Chapter 5
Quantum Wavelet Transforms

ABSTRACT

The classical wavelet transform has been widely applied in the information processing field. It implies that quantum wavelet transform (QWT) may play an important role in quantum information processing. This chapter firstly describes the iteration equations of the general QWT using generalized tensor product. Then, Haar QWT (HQWT), Daubechies D4 QWT (DQWT), and their inverse transforms are proposed respectively. Meanwhile, the circuits of the two kinds of multi-level HQWT are designed. What's more, the multi-level DQWT based on the periodization extension is implemented. The complexity analysis shows that the proposed multi-level QWTs on 2n elements can be implemented by O(n3) basic operations. Simulation experiments demonstrate that the proposed QWTs are correct and effective.

INTRODUCTION

The classical wavelet transform has been widely spread to the information processing field (Mallat, 1989), such as image encryption (Belazi, El-Latif, Diaconu, Rhouma, & Belghith, 2017), image watermarking (Makbol, Khoo, Rassem, & Loukhaoukha, 2017). Its quantum versions, such as Haar quantum wavelet transform (HQWT) and Daubechies D4 quantum wavelet transform (D4QWT), have been proposed (Hoyer, 1997; Fijan & Williams, 1998; Terraneo & Shepelyansky, 2003; Li, Fan, Xia, & Song, 2019). Complexities of HQWT and D4QWT on 2^n elements are $O(n^2)$ and $O(n^3)$, respectively. In contrast, the classical fast wavelet transform needs $O(2^n)$ basic operations

DOI: 10.4018/978-1-7998-3799-2.ch005

to implement the discrete wavelet transform (Beylkin, Coifman, & Rokhlin, 1991). Therefore, quantum wavelet transform achieves exponentially speed up in comparison with its classical counterpart.

This chapter introduces the general quantum wavelet transform, multi-level HQWT and multi-level D4QWT (Li, Fan, Xia, Song, & He, 2018).

GENERAL QUANTUM WAVELET TRANSFORM

Suppose that $W_{2^n}^0 = W_{2^n}$ is a kernel matrix of the general wavelet, then, the (k+1)-level iteration of discrete wavelet transform is defined by (Ruch & Van Fleet, 2011),

$$Y_{2^n}^k = W_{2^n}^k W_{2^n}^{k-1} \dots W_{2^n}^1 W_{2^n}^0, \tag{6.1}$$

where the iteration matrix $W_{2^n}^j$ is

$$W_{2^n}^j = Diag(W_{2^{n-j}}, I_{2^{n-j}}, I_{2^{n-j+1}}, \dots, I_{2^{n-1}}), \tag{6.2}$$

and

$$Diag(W_{2^{n-j}}, I_{2^{n-j}}, I_{2^{n-j+1}}, \dots, I_{2^{n-1}})$$

is a matrix with blocks $W_{2^{n-j}}$, $I_{2^{n-j}}, I_{2^{n-j+1}}, \dots, I_{2^{n-1}}$ on the main diagonal and zeros elsewhere. I_{2^m} is a $2^m \times 2^m$ identity matrix

The iteration equations of $W_{2^n}^j$ and $Y_{2^n}^k$ can be written as

$$W_{2^n}^j = Diag(W_{2^{n-1}}^{j-1}, I_{2^{n-1}}), \tag{6.3}$$

and

$$Y_{2^n}^k = Diag(Y_{2^{n-1}}^{k-1}, I_{2^{n-1}}) W_{2^n}. \tag{6.4}$$

From (5.12), we have