Chapter 16
Coupon Bond Duration and Convexity Analysis:
A Non-Calculus Approach

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ABSTRACT

Coupon bond duration and convexity are the primary risk measures for bonds. Given their importance, there is abundant literature covering their analysis, with calculus being used as the dominant approach. On the other hand, some authors have treated coupon bond duration and convexity without the use of differential calculus. However, none of them provided a complete analysis of bond duration and convexity properties. Therefore, this chapter fills in the gap. Since the application of calculus may be complicated or even inappropriate if the functions in question are not differentiable (as indeed is the case with the bond duration and convexity functions), in this chapter the properties of bond duration and convexity functions by using elementary algebra only are proved. This provides an easier way of approaching this problem, thus making it accessible to a wider audience not necessarily familiar with tools of mathematical analysis. Finally, the properties of these functions are illustrated by using empirical data on coupon bonds.

DOI: 10.4018/978-1-7998-5083-0.ch016

INTRODUCTION

Macaulay's duration and convexity are the most widely used measures of risk both in financial theory and practice. Hence, it is not surprising that the literature is abundant with various titles describing the principals and properties of these two measures. In literature, the common approach to analyzing the properties of Macaulay's coupon bond duration and bond convexity is to use differential calculus of real functions with several real variables (see for example (Aljinović, Marasović & Šego, 2011), (Choudhry, 2001), (Fabozzi, 2006), (Fabozzi, 2007), (Fabozzi & Manu, 2010), (Fabozzi & Manu, 2012), (La Grandville, 2000), (La Grandville, 2001), (Hawawini, 1982), (Hawawini, 1984), (Orsag, 2011) (Pianca, 2005), (Smith, 1998), (Zipf, 2003)). On the other hand, there are also papers which approach the problem of the bond's price properties by using a student-friendly approach via non-calculus (for example (Lawrence & Shankar, 2007) and (Malkiel, 1962)). The main obstacle to using calculus in the analysis of the Macaulay's coupon bond duration and bond convexity is the fact that duration and convexity with respect to coupon-rate (ceteris paribus), yield to maturity (ceteris paribus) or maturity (ceteris paribus) are not continuous functions, not to mention differentiable. In practice, coupon rate, yield to maturity and maturity are discrete variables, so the duration and convexity have to be seen as sequences of real numbers rather then continuous functions. For that reason, Gardijan, Kojić & Šego (2012), Kojić & Lukač (2015), and Kojić & Lukač (2017) have presented the analysis of the Macaulay's coupon bond duration properties by using a non calculus approach. Šego & Škrinjarić (2014) analysed bond convexity properties without using calculus, however in their analysis they didn't take into account the fact that whenever there is a change in any variable of convexity, the corresponding bond price is changing too. This resulted in an incomplete non-calculus analysis of bond convexity properties. To the best of our knowledge, the complete rigorous non-calculus analysis of coupon bond convexity properties is missing in the relevant literature on finance. Therefore, this chapter aims to fill in the missing gap by presenting the complete analysis here. The structure of this chapter is as follows. After the introduction, the second section presents notation used in this study. The third section presents the properties and proofs of bond's price. The non-calculus analysis of Macauly's coupon bond duration and bond convexity is given in the fourth and the fifth section, respectively. The chapter ends with final conclusions in the sixth section.

Notation

In this chapter, the following coupon bond notation is used:

- N: face value, bond's par value
- I: contractual interest rate, bond's coupon-rate
- i: annual coupon payment, a year interest payment
- n: bond maturity, number of payments, n years
- k: annual yield to maturity of the bond
- $r=1+k$: annual period discount factor at rate k
- P: (market) bond's price
- $P(k)$: market price bond as a function of k ceteris paribus
- $P(i)$: market price bond as a function of i ceteris paribus
- $P(n)$: market price bond as a function of n ceteris paribus
- D: Macaulay's bond duration

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