Chapter 6
The Traveling Salesman Problem:
Network Properties, Convex Quadratic Formulation, and Solution

Elias Munapo
North-West University, South Africa

ABSTRACT

The chapter presents a traveling salesman problem, its network properties, convex quadratic formulation, and the solution. In this chapter, it is shown that adding or subtracting a constant to all arcs with special features in a traveling salesman problem (TSP) network model does not change an optimal solution of the TSP. It is also shown that adding or subtracting a constant to all arcs emanating from the same node in a TSP network does not change the TSP optimal solution. In addition, a minimal spanning tree is used to detect sub-tours, and then sub-tour elimination constraints are generated. A convex quadratic program is constructed from the formulated linear integer model of the TSP network. Interior point algorithms are then applied to solve the TSP in polynomial time.

INTRODUCTION

The traveling salesman problem (TSP) was believed to be difficulty until now (Munapo, 2020) or (Papadimitriou, 1977) or (Berman & Karpinski, 2006). The TSP has so many applications which include scheduling, sequencing, vehicle routing, engineering, electronics and genetics. In this paper it is shown that adding or subtracting a constant to all arcs with special features in a traveling salesman problem (TSP) does not change an optimal solution of the (TSP). It is also shown that adding or subtracting a constant to all arcs emanating from the same node in a TSP network does not change the TSP optimal solution. A minimal spanning tree (Wolsey, 1989) is used to detect sub-tours and then sub-tour elimination constraint generated (Bektas & Gouveira, 2006). A convex quadratic program (Jensen & Bard, 2003) is constructed from the formulated linear TSP integer model. Interior point algorithms (Gondzio, 2012) are then applied to solve the TSP in polynomial time.

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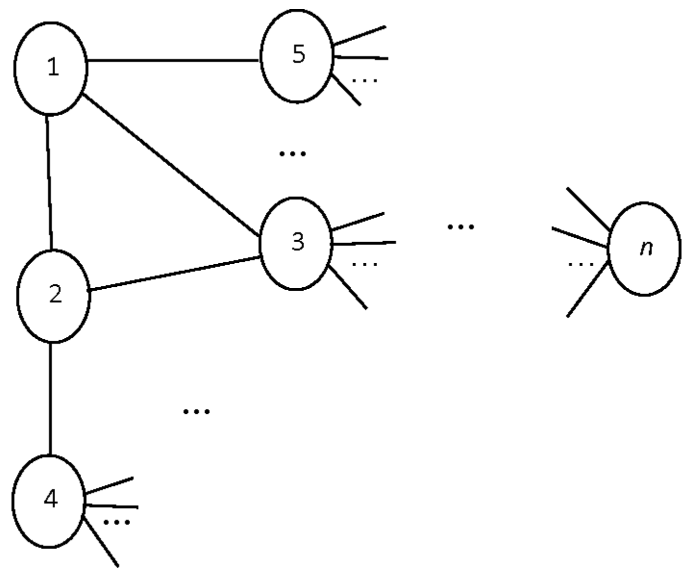
The TSP Model

The objective is to start from an origin node and return to it, traversing the nodes in such a way that every node is visited once and that the total distance travelled is minimized (Applegate et al. 2006) or (Gutin & Punnen, 2006). It is also assumed that all nodes have at least two arcs emanating from them.

Figure 1. TSP model



AVAILABLE METHODS

Exact Approaches

There are several exact approaches that are currently available for the TSP. These include:

a) Brute force. In this approach we try all possible routes and see which one is the cheapest and the worst case complexity is $O(n!)$ and is not practical for large number of nodes.
b) Linear programming based branch, cut and price methods. See Padberg & Rianld (1991), Mitchell (2001), or Nadef (2002) for more on branch and cut methods.
c) Branch and bound method in which assignment sub-problems are used (Winston, 2004).
d) Dynamic programming techniques (Winston, 2004).

A lot of effort has been injected into these exact approaches and a consistent efficient method is now available.