Chapter 7.3
Mathematical Knowledge Management

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INTRODUCTION

Mathematical knowledge is significantly different from other kinds of knowledge. It is abstract, universal, highly structured, extraordinarily interconnected, and of immense size. Managing it is difficult and requires special techniques and tools.

Mathematicians have developed (over the last two or three millennia) many techniques for managing mathematical knowledge. For example, there is a large collection of techniques based on the use of special symbols and notations. Although these techniques are quite effective and have greatly advanced mathematical practice, they are primitive in the sense that the only tools they require are pencil and paper, typesetting machines, and printing presses.

Today mathematics is in a state of transition. Mathematicians are using the Internet in new ways to find information and to share results. Engineers and scientists are producing new kinds of mathematical knowledge that is oriented much more to practical concerns than to theoretical interests. This is particularly true in the field of software development where software specifications and code are forms of mathematical knowledge. Computers are being pushed to perform more sophisticated computations and to mechanize mathematical reasoning. Mathematical knowledge, as a result, is being produced and applied at an unprecedented rate.

It is becoming increasingly difficult to effectively disseminate mathematical knowledge, and to ascertain what mathematical results are known and how they are related to each other. Traditional ways of managing mathematical knowledge are no longer adequate, and current computer and communication technology do not offer an immediate solution. Since mathematical knowledge is vital to science and technology, and science and technology is vital to our society, new ways of managing mathematical knowledge based on new technology and new theory are needed.

This article introduces the main issues of managing mathematical knowledge. It is organized as follows. The Background section describes mathematics as a process of creating, exploring, and connecting mathematical models. The special characteristics of mathematical knowledge and the four main activities that constitute the management of mathematical knowledge are discussed in the Main Focus of the Article. The Future Trends section introduces Mathematical Knowledge Management (MKM), a new field of research, and discusses some of the challenges it faces. The article ends with a conclusion, references, and a list of key terms.

The management of mathematical knowledge is an emerging field of research. Researchers are just starting to build a foundation for it. This article focuses on the core concerns of the field. Except for a few remarks, it does not discuss the parallels between mathematical knowledge management and mainstream knowledge management. Nor does it discuss how techniques for managing mathematical knowledge can be applied to the management of other kinds of knowledge. These are important topics for future research.

BACKGROUND

People often associate mathematics with a body of knowledge about such things as numbers, spatial relationships, and abstract structures. However, this view of mathematics is misleading. It suggests that mathematics is something static and dead, but mathematics is actually the opposite—dynamic and alive. It is more productive and accurate to view mathematics as a process for comprehending the world that consists of three intertwined activities (Farmer & von Mohrenschildt, 2003).

The first activity is the creation of mathematical models that represent mathematical aspects of the world. Mathematical models come in many forms. A well-known and important example is the model of real number arithmetic composed of the set of real numbers, and operations and relations involving the real numbers such as $+$, $\times$, and $<$. Real number arithmetic includes various submodels such as arithmetic of the natural numbers 0,1,2,... and arithmetic of the rational numbers like 2/3, 31/17, and so forth. Real number arithmetic and its submodels capture the essential elements of counting, measurement, motion, and much more. Real number arithmetic itself is a submodel of complex number arithmetic and many other mathematical models.

The second activity is the exploration of mathematical models to learn what they say about the mathematical aspects of the world they model. There are several means of exploration. The explorer can state a conjecture about a model and then attempt to prove that the conjecture is true by virtue of being a logical consequence of the defining properties of the model. The explorer can also formulate a problem concerning the model and then compute a solution to it by mechanically manipulating a representation of the problem using rules determined by the model. A third approach, which is sometimes very effective, is to visualize some facet of the model with a diagram, picture, or animation.

The last activity is the connection of mathematical models by identifying and recording relationships between models. Models can be related to one another in various ways. Examples includes two models being equivalent in a certain sense, one model containing another as a submodel, and one model generalizing another model. A collection of interconnected models facilitates the creation and exploration of new models. New models can be built from old models, and then the results about the old models can be applied to these new models according to how

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