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## Chapter X

# Constraint Satisfaction for Planning and Scheduling

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## ABSTRACT

*As the current planning and scheduling technologies are coming together by assuming time and resource constraints in planning or by allowing introduction of new activities during scheduling, the role of constraint satisfaction as the bridging technology is increasing and so it is important for researchers in these areas to understand the underlying principles and techniques. The chapter introduces constraint satisfaction technology with emphasis on its applications in planning and scheduling. It gives a brief survey of constraint satisfaction in general, including a description of mainstream solving techniques, that is, constraint propagation combined with search. Then, it focuses on specific time and resource constraints and on search techniques and heuristics useful in planning and scheduling. Last but not least, the basic approaches to constraint modelling for planning and scheduling problems are presented.*

## CONSTRAINT SATISFACTION IN A NUTSHELL

Constraint satisfaction is a technology for modelling and solving combinatorial (optimisation) problems. The technology arose from research in artificial intelligence in the mid-1970s and recently it became very popular, especially in the area of scheduling. The basic idea behind constraint satisfaction is to describe the problem declaratively by means of variables and constraints and then to apply generic solving techniques to find an assignment of values to the variables satisfying the constraints. Formally, a *constraint satisfaction problem (CSP)* is a triple $\Theta = (V,D,C)$, where:

- $V = \{v_1,v_2,\ldots,v_n\}$ is a finite set of variables,
- $D = \{D_1,D_2,\ldots,D_n\}$ is a set of domains (i.e., $D_i$ is a set of possible values for the variable $v_i$),
- $C = \{c_1,c_2,\ldots,c_m\}$ is a finite set of constraints restricting the values that the variables can simultaneously take, that is, a constraint is a subset of the Cartesian product of the domains of the constrained variables; we denote by $var(c)$ the set of variables constrained by $c$.

A *solution* to the constraint satisfaction problem $\Theta$ is a complete assignment of the variables from V that satisfies all the constraints. The values for the variables are chosen from their respective domains. Formally, an assignment is a set of pairs *variable/value* such that a given *variable* appears at most once in this set. A complete assignment for V contains a value for every variable from V.

**Example 1 (CSP):** Let $V = \{a,b,c\}$ be a set of variables with domains $D = \{D_a=\{1,2\}, D_b=\{1,2,3\}, D_c=\{2,3\}\}$ and $C = \{a\neq c, a<b, b\neq c\}$ be a set of constraints. Then the following two complete assignments of the variables are (the only) solutions of CSP $\Theta=(V,D,C)$:

- $\alpha_1 = \{a/1, b/2, c/3\}$,
- $\alpha_2 = \{a/1, b/3, c/2\}$.

In pure constraint satisfaction, the domains of variables are assumed to be finite discrete sets of values so the constraint satisfaction problem is a combinatorial problem. Sometimes, a single domain D for all the variables is used (union of all $D_i$). Then the particular domain $D_i$ is specified via a unary constraint over $v_i$.

If domain D consists of exactly two elements then we are speaking about *Boolean constraint satisfaction problems*. Note that an arbitrary CSP can be converted to an equivalent Boolean CSP via a SAT encoding. For example, the variable $x$ with domain of size $n$ can be represented by $n$ Boolean variables indicating the value assigned to the variable $x$; plus there must be constraints between these Boolean variables specifying that exactly one of them has the value *true* and the others have the value *false*. Also, the original constraints over $x$ must be reformulated to use the new Boolean variables instead of $x$.

If there are only binary constraints in C (and perhaps unary constraints to specify domains) then we are speaking about *binary constraint satisfaction problems*. Again, an arbitrary CSP can be converted to an equivalent binary CSP. For example, the role of variables and constraints can be swapped, that is, an n-ary constraint is represented as a variable with domain containing n-tuples satisfying the constraint. This variable is called a dual variable for the constraint. Two dual variables are connected by a binary constraint if the original constraints share a variable. A binary CSP can be naturally

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