Chapter 67
Data Extrapolation via Curve Modeling in Analyzing Risk:
Value Anticipation for Decision Making

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ABSTRACT

Risk analysis needs suitable methods of data extrapolation and decision making. Proposed method of Hurwitz-Radon Matrices (MHR) can be used in extrapolation and interpolation of curves in the plane. For example quotations from the Stock Exchange, the market prices or rate of a currency form a curve. This chapter contains the way of data anticipation and extrapolation via MHR method and decision making: to buy or not, to sell or not. Proposed method is based on a family of Hurwitz-Radon (HR) matrices. The matrices are skew-symmetric and possess columns composed of orthogonal vectors. The operator of Hurwitz-Radon (OHR), built from these matrices, is described. Two-dimensional data are represented by the set of curve points. It is shown how to create the orthogonal and discrete OHR and how to use it in a process of data foreseeing and extrapolation. MHR method is interpolating and extrapolating the curve point by point without using any formula or function.

INTRODUCTION

Method of Hurwitz - Radon Matrices (MHR), invented by the author, can be applied in reconstruction and interpolation of curves in the plane. The method is based on a family of Hurwitz - Radon (HR) matrices. The matrices are skew - symmetric and possess columns composed of orthogonal vectors. The operator of Hurwitz - Radon (OHR), built from these matrices, is described. Author explains how to create the orthogonal and discrete OHR and how to use it in a process of curve interpolation and two-dimensional data modeling. Proposed method needs suitable choice of nodes, i.e. points of the 2D curve to be interpolated or extrapolated: nodes should be settled at each extremum (minimum or maximum) of one coordinate and at least one point between two successive local extrema, and nodes should be

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monotonic in one of coordinates (for example equidistance). Created from the family of N - 1 HR matrices and completed with the identical matrix, system of matrices is orthogonal only for vector spaces of dimensions $N = 1, 2, 4$ or 8. Orthogonality of columns and rows is very important and significant for stability and high precision of calculations. MHR method is modeling the curve point by point without using any formula of function. Main features of MHR method are: accuracy of curve reconstruction depending on number of nodes and method of choosing nodes, interpolation of L points of the curve is connected with the computational cost of rank $O(L)$, MHR interpolation is not a linear interpolation (Ullman & Basri, 1991). The problem of curve length estimation is also considered. Algorithm of MHR method and the examples of data extrapolation are described. Value anticipation is the crucial feature in risk analyzing and decision making.

BACKGROUND

The following question is important in mathematics and computer science: is it possible to find a method of curve interpolation and extrapolation in the plane without building the interpolation polynomials? This chapter aims at giving the positive answer to this question. Current methods of curve interpolation are based on classical polynomial interpolation: Newton, Lagrange or Hermite polynomials and spline curves which are piecewise polynomials (Dahlquist & Bjoerck, 1974). Classical methods are useless to interpolate the function that fails to be differentiable at one point, for example the absolute value function $f(x) = |x|$ at $x = 0$. If point (0;0) is one of the interpolation nodes, then precise polynomial interpolation of the absolute value function is impossible. Also when the graph of interpolated function differs from the shape of polynomials considerably, for example $f(x) = 1/x$, interpolation is very hard because of existing local extrema of polynomial. We cannot forget about the Runge's phenomenon: when interpolation nodes are equidistance then high-order polynomial oscillates toward the end of the interval, for example close to -1 and 1 with function $f(x) = 1/(1+25x^2)$ (Ralston, 1965). MHR method is free of these bad feature. Computational algorithm is considered and then it is important to talk about time. Complexity of calculations for one unknown point in Lagrange or Newton interpolation based on n nodes is connected with the computational cost of rank $O(n^2)$. Proposed method has lower calculation complexity.

A significant problem in risk analysis and decision making is that of appropriate data representation and extrapolation (Brachman & Levesque, 2004). Two-dimensional data can be treated as points on the curve. Classical polynomial interpolations and extrapolations (Lagrange, Newton, Hermite) are useless for data anticipation, because the stock quotations or the market prices represent discrete data and they do not preserve a shape of the polynomial. Also Richardson extrapolation has some weak sides concerning discrete data. This chapter is dealing with the method of data foreseeing and value extrapolation by using a family of Hurwitz-Radon matrices. The quotations, prices or rate of a currency, represented by curve points, consist of information which allows us to extrapolate the next data and then to make a decision (Fagin et al, 1995).

If the probabilities of possible actions are known, then some criteria are ready to be applied in decision making and analyzing risk: for example criterion of Laplace, Bayes, Wald, Hurwicz, Savage, Hodge-Lehmann (Straffin, 1993) and others (Watson, 2002). But in this chapter author considers only two possibilities: to do something or not. For example to buy a share or not, to sell a currency or not. Proposed method of Hurwitz-Radon Matrices (MHR) is used in data extrapolation and then calculations for risk analyzing and decision making are described. MHR method presents new approach to extrapolation problem because it takes the interpolation nodes to create orthogonal basis as columns

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