Chapter 20
Energy of m-Polar Fuzzy Digraphs

Muhammad Akram
University of the Punjab, Pakistan

Danish Saleem
University of the Punjab, Pakistan

Ganesh Ghorai
Vidyasagar University, India

ABSTRACT

In this chapter, firstly some basic definitions like fuzzy graph, its adjacency matrix, eigenvalues, and its different types of energies are presented. Some upper bound and lower bound for the energy of this graph are also obtained. Then certain notions, including energy of m-polar fuzzy digraphs, Laplacian energy of m-polar fuzzy digraphs and signless Laplacian energy of m-polar fuzzy digraphs are presented. These concepts are illustrated with several example, and some of their properties are investigated.

INTRODUCTION

After the introduction of fuzzy sets by Zadeh (1965), fuzzy set theory has been applied in many research fields. Zhang (1994) presented the concept of bipolar fuzzy sets by depicting a positive degree of membership and negative degree of membership, which is an extension of fuzzy sets. This idea has been utilized in different ways, including preference modeling, multi-criteria decision inquiry and cooperative games. The positive degree of membership and negative degree of membership of bipolar fuzzy sets is lying in the interval [-1,1]. The degree of membership 0 of a component in a bipolar fuzzy set implies that the component is inapplicable to the property (K. M. Lee, 2000), the positive degree of membership (0,1] of a component argue that the component obviously satisfies the corresponding property and the negative degree of membership [-1,0) of a component argue that the component apparently satisfies the counter-property. Chen et al. (2014) introduced m-polar fuzzy sets as generalization of bipolar fuzzy

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sets. He demonstrated that bipolar fuzzy sets and 2-polar fuzzy sets are cryptomorphic mathematical notions and that we can acquire briefly one from the corresponding one. The concept back of this is that "multipolar information" (not just bipolar information which correspond to two-valued logic) exists because the data for the real world problems is sometimes taken from n agents $(n \geq 2)$. For example, the exact degree of elecommunication safety of mankind is a point in $[0,1]^n$ ($n \approx 7 \times 10^9$) because different persons have been monitored different times. There are many other examples such as truth degrees of a logic formula which are based on n logic implication operators ($n \geq 2$), similarity degrees of two logic formulas which are based on n logic implication operators ($n \geq 2$), ordering results of a magazine, ordering results of a university, and inclusion degrees (accuracy measures, rough measures, approximation qualities, fuzziness measures, and decision preformation evaluations) of a rough set.

Graph theory was originated by Euler in 1736, when he solved the problem of the Königsberg's bridges, and presented his first paper on graph theory. The energy of a graph firstly proposed by Gutman (1978) and has a vast circle of applications in various fields, including computer science, physics, chemistry and other branch of mathematics. The upper and lower bound for the energy of a graph are discussed by several researchers (Brualdi, 2006; Gutman, 2001; Praba et al. 2014). The concept of fuzzy graphs was introduced by Kaufmann (1973) based on fuzzy relations of Zadeh (1971). Rosenfeld (1975) developed the structure of fuzzy graphs. Anjali and Mathew (2013) examined the energy of a fuzzy graph. Praba proposed the energy of an intuitionistic fuzzy graphs. Akram and Naz (2018) introduced the energy of Pythagorean fuzzy graphs. Naz et al. (2018) discussed some notions of energy in single-valued neutrosophic graphs. Naz et al. (2018a) proposed the notion of energy of bipolar fuzzy graphs. Akram and Waseem (2016) proposed the notion of m-polar fuzzy graphs and Ghorai and Pal (2015, 2016) described several operations and properties on it. Ghorai and Pal (2016a, 2016b) introduced m-polar fuzzy planar graphs, faces and dual of m-polar fuzzy planar graphs. Ghorai and Pal (2016c) also defined isomorphism and complement of m-polar fuzzy graphs. In this chapter, we present certain notions, including energy of m-polar fuzzy digraphs, Laplacian energy of m-polar fuzzy digraphs and signless Laplacian energy of m-polar fuzzy digraphs. We investigate some of their properties. For other notations, terminologies and applications not mentioned in the paper, the readers are referred to Akram (2019).

BACKGROUND

In this section, we review basic definitions which are helpful for later sections.

Definition 2.1 (Anjali and Mathew, 2013) An ordered pair of sets G = (V,E) where V is a nonempty finite set and E consisting of 2-element subsets of elements of V is called a graph. It is denoted by G = (V,E). V is called vertex and edge set respectively. The elements in V and E are called vertices and edges respectively.

Definition 2.2 A pair of the form $D = (V, E)$ is called directed graph, where V is a set whose elements are called vertices, nodes, or points and E is a set of ordered pairs of vertices, called arrows, directed edges, directed arcs, or directed lines.

Definition 2.3 (Gutman, 1978, 2001) Let $G = (V, E)$ be a graph. The energy of G is defined as the sum of its absolute eigenvalues, i.e.,