Chapter 18
Recent Developments on the Basics of Fuzzy Graph Theory

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ABSTRACT

In this chapter, the authors introduce some basic definitions related to fuzzy graphs like directed and undirected fuzzy graph, walk, path and circuit of a fuzzy graph, complete and strong fuzzy graph, bipartite fuzzy graph, degree of a vertex in fuzzy graphs, fuzzy subgraph, etc. These concepts are illustrated with some examples. The recently developed concepts like fuzzy planar graphs are discussed where the crossing of two edges are considered. Finally, the concepts of fuzzy threshold graphs and fuzzy competitions graphs are also given as a generalization of threshold and competition graphs.

INTRODUCTION

The problem of "Seven bridges of Koningsberg" gives birth to a new dimension of mathematics. In late 1730, Leohhard Euler solved the problem using a model named as "Graph". Since then, graphs are being used as a mathematical tool for modeling real world problems. Graphs can be used in the situation where the objects are connected by some rules or relations. For instance, tramway where tram depots are connected by roads, circuit design where nodes are connected by wires, etc. Nowadays, graphs are being used in games and recreational mathematics too. But, the demand of real world does not end here. There are a lot of uncertainty in the real world. One of them is vagueness or fuzziness. Thus researchers are now studying a new branch of mathematics called "Fuzzy graph". Rosenfeld (1975) first considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs. Many real life applications such as time scheduling, networking, communication, image segmentation, data mining, etc. can be solved by fuzzy graphs.

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The definition of a fuzzy graph is given below.

BACKGROUND

A *fuzzy graph* (Rosenfeld, 1975) $\xi = (V, \sigma, \mu)$ of the graph $G^* = (V, E)$ is a non-empty set V together with a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ such that for all $x, y \in V$,

$$\mu(x,y) \leq \min\{\sigma(x), \sigma(y)\},$$

where $\sigma(x)$ and $\mu(x,y)$ represent the membership values of the vertex x and of the edge (x,y) in ξ respectively. A loop at a vertex x in a fuzzy graph is represented by $\mu(x,x) \neq 0$. An edge is non-trivial if $\mu(x,y) \neq 0$.

An example of fuzzy graph is given Figure 1.

In Figure 1, u, v, w, s, t are the vertices with membership values 0.7, 0.6, 0.8 respectively. There are 6 edges $(u,v), (u,w)$, (v,w), (w,s), (w,t) and (s,t) with membership values 0.5, 0.7, 0.6, 0.7, 0.8 and 0.5 respectively.

A fuzzy graph can be drawn in many different ways by changing the positions of the vertices and drawing the edges by straight lines or curved lines. It is immaterial whether the edges are drawn straight line or curved, short or long. The important thing is the relationship among the vertices.

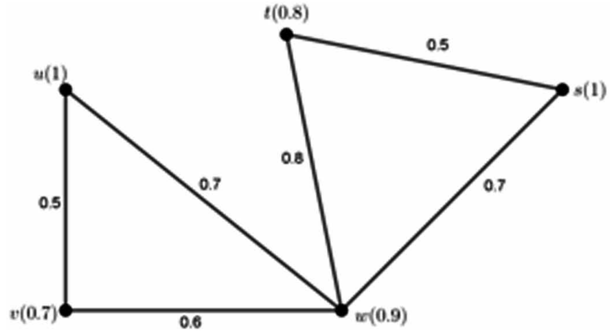
If the vertex set V and edge set E are finite then the graph is called a finite graph otherwise, it is called infinite graph. In general, a graph means a finite graph.

Directed and Undirected Fuzzy Graphs

Directed fuzzy graphs (or simply fuzzy digraph) are the fuzzy graphs in which the fuzzy relations between edges are not necessarily symmetric. The definition of directed fuzzy graph is as follows:

Definition 2.1 (Mordeson and Nair, 1996) *Directed fuzzy graph (fuzzy digraph)* $\vec{\xi} = (V, \sigma, \vec{\mu})$ *is a non-empty set V together with a pair of functions* $\sigma : V \to [0,1]$ *and* $\vec{\mu} : V \times V \to [0,1]$ *such that for all* $x, y \in V$, $\vec{\mu}(x,y) \leq \sigma(x) \wedge \sigma(y)$.

Figure 1. An example of fuzzy graph



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