

Chapter 10

Tripartite and Quadpartite Size Ramsey Numbers for All Pairs of Connected Graphs on Four Vertices

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ABSTRACT

A popular area of graph theory is based on a paper written in 1930 by F. P. Ramsey titled “On a Problem on Formal Logic.” A theorem which was proved in his paper triggered the study of modern Ramsey theory. However, his premature death at the young age of 26 hindered the development of this area of study at the initial stages. The balanced size multipartite Ramsey number $m_j(H, G)$ is defined as the smallest positive number s such that $K_j \times s \rightarrow (H, G)$. There are 36 pairs of (H, G) , when H, G represent connected graphs on four vertices (as there are only 6 non-isomorphic connected graphs on four vertices). In this chapter, the authors find $m_j(H, G)$ exhaustively for all such pairs in the tripartite case $j=3$, and in the quadpartite case $j=4$, excluding the case $m_4(K_4, K_4)$. In this case, the only known result is that $m_4(K_4, K_4)$ is greater than or equal to 4, since no upper bound has been found as yet.

HISTORY

After the publication of the original paper by F. P. Ramsey, the resurrection of Ramsey Theory for graphs emerged in the paper by Paul Erdős and George Szekeres, published around 1935.

Theorem (by Paul Erdős and George Szekeres)

For any $m \geq 2$ and $n \geq 2$, $r(m, n)$ exists and it satisfies

$$r(m, n) \leq r(m-1, n) + r(m, n-1)$$

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and

$$r(m, n) \leq \binom{m+n-2}{m-1}.$$

Following the publication of the paper by Paul Erdős and George Szekeres, the exact determination of these numbers for small graphs has been attempted by many mathematicians. Unfortunately, their progress was hindered by the stubbornly resistant $(r(5, 5))$, which is presently known to lie between 43 and 48 (see Vigleik Angeltevit and Brendan D. McKay (Angeltevit, 2017) for the best upper bound of 48 and Exoo (Exoo, 1989) for the best lower bound of 43). Finding sharp bounds for $(r(6, 6))$ appears to be an even more arduous task. This has been eloquently expressed by the great mathematician Paul Erdős using the following quotation (see 1990 Scientific American article by Ronald Graham and Joel Spencer). *Suppose aliens invade the earth and threaten to obliterate it in a year's time unless human beings can find the Ramsey number for red five and blue five $(r(5, 5))$. We could marshal the world's best minds and fastest computers, and within a year we could probably calculate the value. If the aliens demanded the Ramsey number for red six and blue six $(r(6, 6))$, however, we would have no choice but to launch a pre-emptive attack.* – Paul Erdős (1990 Scientific American article by Ronald Graham & Joel Spencer)

Researchers are now trying to approach this problem by using new techniques to investigate whether the lower bound 43 (Exoo, 1989) and the upper bound 48 (Angeltevit (2017)) could be improved. Offshoots of classical Ramsey numbers, introduced by Burger and Vuuren (2004), Syfrizal et al., (2005, 2012), are the multipartite Ramsey numbers.

Let K_N denote the complete graph on N vertices. For any two graphs, say H, G (without loops and parallel edges), we say that $K_N \rightarrow (H, G)$, if for any red/blue colouring of K_N , given by $K_N = H_R \oplus H_B$, there exists a red copy H in H_R or a blue copy G in H_B . In Ramsey theory one important part deals with the exact determination of Ramsey numbers, $r(H, G)$, defined as the smallest positive number N such that $K_N \rightarrow (H, G)$. Formally, the balance multipartite graph $K_{j \times s}$ can be viewed as a graph consisting of the vertex set

$$V(K_{j \times s}) = \{v_{m,n} : m \in \{1, 2, \dots, j\} \text{ and } n \in \{1, 2, \dots, s\}\}$$

and the edge set

$$E(K_{j \times s}) = \{(v_{m,n}, v_{m',n'}) : m, m' \in \{1, 2, \dots, j\}, m \neq m' \text{ and } n, n' \in \{1, 2, \dots, s\}\}.$$

In such a graph, the set of vertices in the m^{th} partite set is denoted by $\{v_{mn} \mid n \in \{1, 2, \dots, s\}\}$. The Ramsey number $m_j(H, G)$ is defined as the smallest positive number s such that that $K_{j \times s} \rightarrow (H, G)$.

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