Chapter 9
Sumset Valuations of Graphs and Their Applications

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ABSTRACT

Graph labelling is an assignment of labels to the vertices and/or edges of a graph with respect to certain restrictions and in accordance with certain predefined rules. The sumset of two non-empty sets A and B, denoted by A+B, is defined by A+B=\{a=b: a\inA, b\inB\}. Let X be a non-empty subset of the set \Z and \sP(X) be its power set. An \textit{sumset labelling} of a given graph G is an injective set-valued function f: V(G)\to\sP_0(X), which induces a function f+: E(G)\to\sP_0(X) defined by f+(uv)=f(u)+f(v), where f(u)+f(v) is the sumset of the set-labels of the vertices u and v. This chapter discusses different types of sumset labeling of graphs and their structural characterizations. The properties and characterizations of certain hypergraphs and signed graphs, which are induced by the sumset-labeling of given graphs, are also done in this chapter.

INTRODUCTION

In graph theory, graph labelling is an assignment of labels to the vertices and/or edges of a graph with respect to certain restrictions and in accordance with certain predefined rules. For the past few decades, graph labelling has become a fruitful research area in graph theory. Different graph labellings have resulted from practical considerations. They are not only of theoretical interests but have many practical implications also. For many applications, the edges or vertices of a given graph are given labels that are meaningful in the context.

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Graph Labelling

Valuation of a graph G is a one to one mapping of the vertex set of G on to the set of all integers $\mathbb{Z}$. Graph labeling has become a fertile research area in graph theory after the introduction of the notion of certain types number valuations, called β-valuations, of graphs in Rosa (1967). The most popular one among the number valuations of graphs is β*-valuation* or *graceful labelling* of a graph (see Golomb, 1972; Rosa, 1967) which is defined as an injective function

$$f : V\left(G\right) \to \left\{1, 2, 3, \ldots, \left|E\right|\right\}$$

such that, when each edge xy is assigned the label $\left|f\left(x\right) - f\left(y\right)\right|$, the resulting edge labels are distinct. A graph $G\left(V, E\right)$ is said to be *edge-graceful* if there exists a bijection

$$f : E \to \left\{1, 2, 3, \ldots, \left|E\right|\right\}$$

such that the induced mapping

$$f^{+} : V \to \left\{0, 1, \cdots, \left|V\right| - 1\right\}$$

given by

$$f^{+}\left(u\right) = f\left(uv\right)\left(\mathrm{mod}\left|V\right|\right)$$

taken over all edges uv is a bijection.

Motivated by various problems in social networks, a set analogue of number valuations called set-valuation of graphs, has been introduced in Acharya (1983). In the number valuations, the elements of a graph are assigned to numbers while in the set-valuations, the elements of a graph are assigned to sets subject to certain conditions. The mathematical definition of a set-valuation of a graph concerned is as given below (see Acharya, 1983):

For a graph G and a non-empty set X, a *set-labeling* or a set-valuation of G, with respect to the set X, is an injective set-valued function

$$f : V\left(G\right) \to \mathcal{P}\left(X\right)$$

such that the function

$$f^{\oplus} : E\left(G\right) \to \mathcal{P}\left(X\right) - \left\{\varnothing\right\}$$