Chapter 2
An Introduction to Intersection Graphs

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ABSTRACT

In this chapter, a very important class of graphs called intersection graph is introduced. Based on the geometrical representation, many different types of intersection graphs can be defined with interesting properties. Some of them—interval graphs, circular-arc graphs, permutation graphs, trapezoid graphs, chordal graphs, line graphs, disk graphs, string graphs—are presented here. A brief introduction of each of these intersection graphs along with some basic properties and algorithmic status are investigated.

INTRODUCTION

The graphs are very useful tool to model a huge number of real life problems starting from science, technology, medical science, social science and many other areas. The geometrical and topological stuctures of any communication system such as Internet, Facebook, Whatsapp, ResearchGate, Twiter, etc. are based on graph. So graph theory is an old as well as young topic of research as till today graphs are used to solve several problems. Depending on the geometrical structures and properties different type of graphs are defined, viz. path, cycle, tree, complete graph, planar graph, perfect graph, chordal graph, tolarence graph, intersection graph, etc.

In this chapter, diferent types of intersection graphs (IntGs) are defined and investigated their properties.

Let $S = \{S_1, S_2, \ldots\}$ be a finite or infinite set of sets. For each set S_i we consider a vertex (v_i) and there is an edge between the vertices v_i and v_j if the corresponding sets $S_i \cap S_j$ have a non-empty intersection. That is, the set of vertices $V = \left\{v_i, v_2, \ldots\right\}$ and the set of edges

$$E = \{(v_i, v_j) : S_i \cap S_j \neq \varphi\}.$$

The resultant graph is called intersection graph.

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Table 1. Table of abbreviations

Abbreviation	Description
IntG	Intersection graph
InvG	Interval graph
CirG	Circular-arc graph
PerG	Permutation graph
TraG	Trapezoidal graph
TolG	Tolerance graph
PCA	Proper circular-arc
UCA	Unit circular arc
GIG	Grid intersection graphs

One important class of graph is perfect graph defined as follows:

A graph (undirected) $G=(V,E)$ is called χ-perfect if for all

$A \subseteq V$, $\omega(G(A))=\chi(G(A))$,

and G is called α-perfect if for all

$A \subseteq V$, $\alpha(G(A))=\kappa(G(A))$,

where $G(A)$ is the induced subgraph by the subset *A*. A graph is said to be *perfect* if it is either χ-perfect or α-perfect. In the famous *perfect graph theorem* is it proved that a graph is χ-perfect if and only if it is α-perfect.

An undirected graph G is said to be *triangulated* if every cycle of length four or more has a chord. This type of graphs are also known as *chordal graphs, monotone transitive graphs, rigid-circuit graphs* and *perfect elimination graphs*.

A clique of a graph $G=\left(V,E\right)$ is a set of vertices $S \subseteq V$ in which all the vertices of *S* are adjecent, i.e. $\left(v_i,v_j\right) \in E$ for all $v_i,v_j \in V$. A *maximal clique* is a special type of clique for which no further vertex can be added so that it remains a clique. That is, if *C* is a maximal clique of the graph $G=\left(V,E\right)$, then $C \cup \left\{v\right\}$ is not a clique for $v \notin C$ and any $v \in V$. A clique is said to be *maximum* if the cardinality is maximum among all others cliques of the graph.

The *clique graph* $C(G)$ of a graph G is the IntG of the family of all cliques of G. *Cographs* are defined as the graphs which can be reduced to single vertices by recursively complementing all connected subgraphs. This graphs are known as complement reducible graphs.

Let $N\left[v\right]\left(N\left(v\right)\right)$ be the closed (open) neighbour of the vertex *v*. A vertex v is called *simplicial* if and only if $N[v]$ is a clique. The ordering $(v_1,v_2,\ldots,v_n)$ of V is called a perfect elimination ordering if and only if for all $i \in \{1,2,\ldots,n\}$ the vertex v_i is simplicial in G_i, where