Chapter 53
Constrained Nonlinear Optimization in Information Science

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ABSTRACT

This chapter provides an overview of constrained optimization methods. Background, theory, and examples are provided. Coverage includes Lagrange multipliers for equality constrained optimization with a Cobb-Douglass example from information science. The authors also provide Karush-Kuhn-Tucker for inequality-constrained optimization and a production example for smart phones with inequalities. An overview and discussion of numerical methods and techniques is also provided. The authors also provide a brief list of technology available to assist in solving these constrained nonlinear optimization problems.

INTRODUCTION

A company manufactures new smart-phones that are supposed to capture the market by storm. The two main inputs components of the new smart-phone are the circuit board and the relay switches that make the phone faster and smarter and give it more memory.

The number of smart-phones to be produced is estimated to equal $E = 200x_1^{\frac{1}{3}}x_2^{\frac{1}{2}}$, where E is the number of smart-phones produced and x_1 & x_2 are the number of circuit broad hours and the number of relay hours worked, respectively. Such a function is known to economists as a *Cobb–Douglas function*. Laborers are paid by the type of work they do: the circuit boards and the relays for \$5 and \$10 an hour, respectively. We want to maximize the number of smart-phones to be made if we have \$150,000 to spend on these components in the short run.

Lagrange multipliers can be used to solve *nonlinear optimization problems* (NLPs) in which all the constraints are equality constrained. We consider the following type of NLPs as shown by Equation (1):

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Maximize (Minimize) $z = f(x_1, x_2, ..., x_n)$ (1)

Subject to

$$g_1(x_1, x_2, ..., x_n) = b_1$$
$$g_2(x_1, x_2, ..., x_n) = b_2$$
$$...$$
$$g_m(x_1, x_2, ..., x_n) = b_m$$

In our smart-phones example, we find we can build an equality constrained model. We want to maximize

$$E = 200 x_1^{\frac{1}{3}} x_2^{\frac{1}{2}}$$

subject to the equality constraint

$$5x_1 + 10x_2 = 150{,}000$$

Problems in information science and technology such as this can be modeled using constrained optimization. We begin our discussion with equality constrained optimization, then discuss the inequality constrained optimization, and finally discuss some numerical methods to approximate the solutions.

BACKGROUND

The general constrained nonlinear programming (NLP) problem is to find x* as to optimize $f(X), X = (x_1, x_2, ..., x_n)$ subject to the constraints of the problem shown in equation (2).

Maximize or Minimize $f(x_1, x_2, ..., x_n)$

subject to

$$g_i(x_1, x_2, ..., x_n) \begin{Bmatrix} \geq \\ = \\ \leq \end{Bmatrix} b_i \qquad (2)$$

for i = 1,2,…,m.

Classical constrained optimization appeared with equality constrains and Lagrange multiplier named for Joseph Lagrange in the late 1700's. It was almost two hundred years later when Kuhn-Tucker (1951) presented their famous Kuhn-Tucker (KT) conditions. Scholars later found that Karsuh (1939) had done

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