Chapter 2
Distributed Parameter Systems Control and Its Applications to Financial Engineering

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ABSTRACT

The chapter analyzes differential flatness theory for the control of single asset and multi-asset option price dynamics, described by PDE models. Through these control methods, stabilization of distributed parameter (PDE modelled) financial systems is achieved and convergence to specific financial performance indices are made possible. The main financial model used in the chapter is the Black-Scholes PDE. By applying semi-discretization and a finite differences scheme the single-asset (equivalently multi-asset) Black-Scholes PDE is transformed into a state-space model consisting of ordinary nonlinear differential equations. For this set of differential equations, it is shown that differential flatness properties hold. This enables one to solve the associated control problem and to stabilize the options' dynamics. By showing the feasibility of control of the single-asset (equivalently multi-asset) Black-Scholes PDE, it is proven that through selected purchases and sales during the trading procedure, the price of options can be made to converge and stabilize at specific reference values.

INTRODUCTION

In several problems of financial engineering, such as options and commodities trading, forecasting of options' values, estimation of financial distress and credit risk assessment, validation of option pricing models, etc. one comes against Partial Differential Equations (PDEs). Moreover, in problems of control of financial systems where the aim is to stabilize financial processes which are described by PDE models, one has to harness again the complex PDE dynamics through the application of an external input. In the recent years differential flatness theory has emerged as an approach to the control and stabilization

of systems described by PDE dynamics (Rudolph, 2003), (Rigatos, 2015). This research work focuses on differential flatness theory for the control and stabilization of single asset and multi-asset option price dynamics, described by PDE models. It is shown how the differential flatness approach achieves, stabilization of distributed parameter financial systems (that is systems modelled by PDEs) and how it enables convergence to specific financial performance indexes (Rigatos, 2014a; Rigatos, 2014b; Rigatos, 2014c; Rigatos, 2015a; Rigatos, 2015b; Rigatos, 2015c).

The Black-Scholes PDE is the principal financial model used in this study. It is demonstrated how with the use of semi-discretization and a finite differences scheme the single-asset (equivalently multi-asset) Black-Scholes PDE is transformed into a state-space model consisting of ordinary nonlinear differential equations. For this set of differential equations it is proven that differential flatness properties hold (Rigatos, 2011; Rigatos, 2013; Rigatos, 2015). This permits to arrive at a solution for the associated control problem and to ascertain stabilization of the options' dynamics. By proving that it is feasible to control the single-asset (equivalently multi-asset) Black-Scholes PDE it is also concluded that through a selected trading policy, the price of options can be made to converge and stabilize at specific reference values.

The computational part of the considered feedback control method is as follows: For the local subsystems, into which the single-asset (equivalently multi-asset) Black-Scholes PDE is decomposed, it becomes possible to apply boundary-based feedback control. The controller design proceeds by showing that the state-space model of the single-asset (equivalently multi-asset) Black-Scholes PDE stands for a differentially flat system. Next, for each subsystem which is related to a nonlinear ODE, a virtual control input is computed, that can invert the subsystem's dynamics and can eliminate the subsystem's tracking error. From the last row of the state-space description, the control input (boundary condition) that is actually applied to the single-asset (equivalently multi-asset) Black-Scholes PDE system is found. This control input contains recursively all virtual control inputs which were computed for the individual ODE subsystems associated with the previous rows of the state-space equation. Thus, by tracing the rows of the state-space model backwards, at each iteration of the control algorithm, one can finally obtain the control input that should be applied to the single-asset (equivalently multi-asset) Black-Scholes PDE system so as to assure that all its state variables will converge to the desirable setpoints.

The structure of the chapter is as follows: in Section "*The problem of boundary control of the single-asset Black- Scholes PDE*", an overview about the single-asset Black-Scholes PDE is given and the associated boundary control problem is formulated. In Section "*Option pricing modelling with the use of the single-asset Black-Scholes PDE*" the concept of option pricing for the single-asset Black-Scholes PDE is explained. In Section "*Transformation of the single-asset Black-Scholes PDE into nonlinear ODEs*" it is explained how the single-asset Black-Scholes PDE dynamics can be transformed to an equivalent state-space form. In Section "*Computation of boundary control for the single-asset Black-Scholes PDE* " a boundary feedback control law is computed for the single-asset Black-Scholes PDE In Section "*Closed loop dynamics of the single-asset Black-Scholes PDE*" the dynamics of the closed control loop of the single-asset Black-Scholes PDE is analysed. In Section "*The problem of boundary control of the multi-asset Black-Scholes PDE* " the multi-asset Black-Scholes PDE is introduced and the associated boundary control problem is formulated. In Section "*Boundary control of the multi-asset Black-Scholes PDE*" it is explained how the multi-asset Black-Scholes PDE can be transformed into an equivalent state-space description. In Section "*Flatness-based control of the multi-asset Black-Scholes PDE*" it is analysed how a boundary feedback control input can be computed for the multi-asset Black-Scholes PDE. In Section "*Simulation tests*" the satisfactory performance of the control loop is confirmed

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