Chapter XIII
About the Point Location Problem

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ABSTRACT

In this chapter we present some well-known algorithms for the solution of the point location problem and for the more particular problem of point-in-polygon determination. These previous approaches to the problem are presented in the first sections. In the remainder of the paper, we present a quick location algorithm based on a quaternary partition of the space, as well as its associated computational cost.

INTRODUCTION

In this chapter the authors present a new approach for the general solution of the point-location problem[1] and the particular solution of the point-in-polygon problem (Preparata, 1985) on which we will focus our attention. Some of the most efficient solutions for the point-in-polygon problem reduce the solution to a solution of other fundamental problems in computational geom-etry, such as computing the triangulation of a polygon or computing a trapezoidal partition of a polygon to solve then, in an efficient way, the point-location problem for that trapezoidal partition. Nevertheless, two different methods for solving the point-in-polygon problem have become popular: counting ray-crossings and computing "winding" numbers. Both algorithms lead to solutions with a less-than-attractive cost of O(n), however the first one is significantly better than

the second (O'Rourke, 2001). An implementation comparison by Haines (1994) shows the second to be more than twenty times slower.

Methods of polygon triangulation include greedy algorithms (O'Rourke, 2001), convex hull differences by Tor and Middleditch (1984) and horizontal decompositions. Regarding solutions for the point-location problem, there is a long history in computational geometry. Early results are surveyed by Preparata and Shamos (1985). Of all the methods suggested for the problem, four basically different approaches lead to optimal O(log n) search time, and O(n) storage solutions. These are the chain method by Edelsbrunner et al. (1986) which is based on segment trees and fractional cascading, the triangulation refinement method by Kirkpatrick (1983), the use of persistency by Sarnak et al. (1986), and the randomised incremental method by Mulmuley et al. (1997). Recent research has gone into dynamic point location, where the subdivision can be modified by adding and deleting edges. A survey on dynamic point location is given by Chiang et al. (1991) . Extension beyond two dimensions of the point-location problem is still an open research topic (Carvalho et al., 1995).

POINT LOCATION PROBLEMS IN SIMPLE CASES

Several algorithms are available to determine whether a point lies within a polygon or not. One of the more universal algorithms is counting ray crossings, which is suitable for any simple polygons with or without holes. The basic idea of the counting ray crossings algorithm is as follows:

Let L be the scan-line crossing the given point, Lcross is the number of left-edge/ray crossings and Rcross is the number of right-edge/ray crossings. First, we need to determine whether the given point is a vertex of the polygon or not; if not, we calculate Lcross and Rcross. If the parities of Lcross and Rcross are different (Figure

1 case B) then this point lies on the edge of the polygon. Otherwise, if Lcross (or Rcross) is an odd number (Figure 1 case A), the point must lie inside the polygon. If Lcross (or Rcross) is an even number (cases C, D, E), the point must lie outside the polygon.

When the polygon is degraded to a triangle, the above-mentioned problem can be solved by the following algorithm shown in Figure 2. We assume the three vertices A, B and C are oriented counterclockwise. If point p lies inside ABC, then P must lie to the left of all three vectors AB, BC and CA. Otherwise, p must lie outside ABC. This algorithm may also be applied to any convex polygon, in principle without holes, if the vertices of the polygon are oriented in counterclockwise or clockwise order.

An equivalent algorithm is based on the inner product among the vector defined by the view point of the query point and the normal vectors of the edges. This algorithm can be directly used for convex polygons, including the degenerate case of a convex polygon with holes in its interior. The main criterion of this algorithm is based on the analysis of the sign of the inner products among the normal vectors of the edges of the polygon, with the unitary vector whose direction is deter-

Figure 1. Ray-crossings algorithm
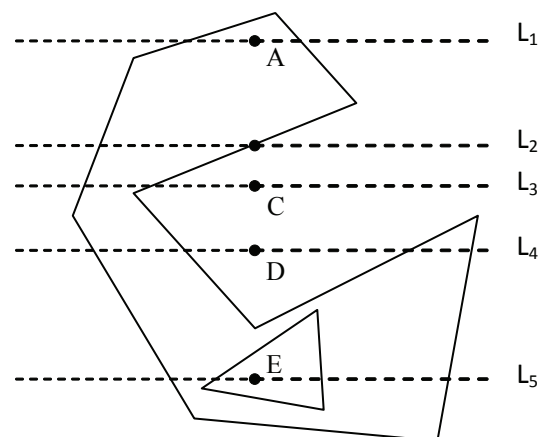
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