Simulating Complex Supply Chain Relationships Using Copulas

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INTRODUCTION

Supply chains involve complex interactions between different aspects of the research question and it is extremely difficult (if not impossible) to derive closed-form analytical solutions, leaving simulation as the only available alternative. The use of simulation in functional areas of business is well documented and includes accounting, finance, marketing, and other areas (Pegden et al., 1995). The analysis of supply chain systems is no exception. Recently, Waller and Fawcett (2012) observed that "…as simulation methods and technology are advanced, metaheuristics exist to find near optimal solutions, and powerful optimization is now in the hands of nearly any manager, let us not just model these simple systems, let us model the realistic, complex systems—in for a penny, in for a pound." The statement "When all else fails, there is simulation!" (Evers and Wan, 2012) is by no means an exaggeration.

The Council of Supply Chain Management Professionals defines supply chain management as encompassing "the planning and management of all activities involved in sourcing and procurement, conversion, and all logistics management activities. Importantly, it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third-party service providers, and customers. In essence, supply chain management integrates supply and demand management within and across companies." When modeling this complex system, simulation may be the best alternative as the analytical tool (Waller and Fawcett, 2012). Swaminathan et al. (1998) also strongly argue for the use of simulation in supply chains.

This is because complex interactions between different entities and the multitiered structure of supply chains make it difficult to utilize closed-form analytical solutions. Benchmarking solutions provide insights into current trends but are not prescriptive. This leaves simulation as the only viable platform for detailed analysis for alternative solutions. Evers and Wan (2012) offer an excellent analogy in support of simulation. Griffis et al (2012), Rogers et al (2012), and Bartolacci et al (2012) also provide convincing evidence of the power of simulation as a methodological tool.

Constructing and modeling supply chains in practice, however, presents many challenges. Supply chains often involve "one or more families of related products", involving different entities or organizations that have different objectives, that are "highly interdependent when it comes to improving the performance of the supply chain in terms of objectives such as on-time delivery, quality assurance, and cost minimization." (Swaminathan et al., 1998). Hence, in order to effectively analyze supply chains using simulation, it is necessary to model the interdependence between different aspects of the model. Such interdependence may manifest itself in the demand for products, in the delivery of raw materials, in set up times, etc. Often, distribution of demand, delivery times, and set up times are not normally distributed, and must

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be described using non-normal distributions such as the Gamma, Weibull, Beta, and other similar skewed distributions (Burgin, 1975; Wagner et al, 2009). Furthermore, it is possible that the demand for a given product has a certain marginal distribution (say Gamma), while that of another has a completely different distribution (say Weibull), but the two demand distributions are related. Thus, in order to analyze the performance and management of supply chains and similar complex systems, the ability to simulate random variates whose marginal distribution and dependence are specified is a necessary pre-requisite.

Currently, we do not have the ability even to completely specify such joint distributions, and the generation of such related random variables is extremely difficult. One possible approach for modeling the joint distribution of a set of variables with non-normal marginal distributions is to use copulas. The use of copulas for modeling complex supply chain relationships is not new. It has been previously used for modeling supplier default dependencies (Wagner et al, 2009) and to estimate the joint distribution of the disruption vector (Masihtehrani, 2011). The objective of this study is to demonstrate the viability of a method based on copulas for generating related random variates with specified marginal distributions and dependence structure for modeling supply chain inter-dependencies.

BACKGROUND

The ease with which random variates can be generated has improved considerably in recent years. This improvement can be attributed to improved hardware and software that is currently available (compared to just 15-20 years ago). Now, even standard desktop software such as Excel offer the ability to perform basic simulation without any other software. Software such as Crystal Ball and @Risk offer the ability to analyze complex decision analysis scenarios, and software such as Arena, GPSS, and ProModel offer the ability to simulate complex systems. One of the crucial aspects of any simulation software is the ability to generate random variates from specified distributions in order to investigate the impact of changes in assumptions on the results of the analysis. Discussion on generating random variates can be found in practically all simulation textbooks. Readers interested in the technical details of algorithms are referred to the book by Fishman (1996), while those interested in the application of simulation are referred to the book by Law (2007).

While most textbooks offer a discussion of the generation of univariate forms of normal and non-normal random variates, discussion of multivariate random variates is invariably limited to the normal distribution. The generation of multivariate normal distribution is well understood and documented and many algorithms are available (see for example Fishman, 1996). Discussion of multivariate non-normal distributions, however, is another matter. Few algorithms for the generations of multivariate non-normal distributions are available due to the general complexity of non-normal multivariate distributions.

A few joint distributions for the bivariate Gamma and other non-normal distributions such as the Weibull and Exponential have been proposed (Kotz et al., 2000). Note that in these cases, it is assumed that the marginal distribution of the individual variables comes from the same family (such as Gamma, Weibull, etc.). Even then, these distributions are extremely complex. There are very few cases where the joint distribution of a set of variables whose marginal distributions are arbitrary (do not come from the same family of distributions) can even be specified. For example, consider a simple case where the marginal distribution of demand for a set of four products is given by the Gamma, the Uniform, the Normal, and Beta distributions, and that the demand for these four products are related. We currently do not even have the ability to specify the joint distribution of demand for the four products with the characteristics above. When the joint distribution is not specified, then it is impossible to derive