# Methods for Improving Alias Rejections in Comb Filters

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## INTRODUCTION

Decimation is the process of decreasing the sampling rate by an integer $M$ in the digital domain. This process is used in sub band coding, filter banks, communication systems, oversampled A/D (analog/digital) converters, among others. If the signal is not appropriately filtered, the unwanted replicas of the main spectrum of the decimated signal, called aliasing, will be present. Therefore, to prevent aliasing in the decimated signal, the signal must be first filtered by a low pass filter, called anti-aliasing, or decimation filter. As a result, the process of decimation consists of two principal stages: filtering and down sampling (decreasing the sampling rate by integer $M$). The integer value $M$ is also called the decimation factor.

In order to avoid high order decimation filters, the decimation is usually performed in two or more stages. The most simple decimation filter is comb filter, (Hogenauer, 1981) which usually works at high input rate. The transfer function of the comb filter is given as:

$$H(z) = \left[ \frac{1}{M} \frac{1 - z^{-M}}{1 - z^{-1}} \right]^K , \quad (1)$$

where $K$ is the order of the comb, and $z$ is a complex variable.

The magnitude characteristic of the comb filter, expressed in digital frequency $\omega$, is given as:

$$\left| H(e^{j\omega}) \right| = \left| \frac{1}{M} \frac{\sin(\omega M / 2)}{\sin(\omega / 2)} \right|^K . \quad (2)$$

The magnitude characteristic must be flat in the pass band of interest. Additionally, in order to eliminate aliasing, the comb filter must have a high attenuation in the so called folding bands, which are the bands around the zeros of the comb filter.

## BACKGROUND

A simple method to increase the attenuation in the comb folding bands consists of increasing the order of the comb filter, as shown in Figure 1. Here, the decimation factor $M$ is 10 and the values of order $K$ are 1, 3, and 5. The zooms in the pass bands in Figure 1 show that the pass band droop increases with the increase of the order of the comb filter.

The objective of this article is to present the methods proposed so far to increase the bandwidths around the comb zeros, and thus increase the comb alias rejection.

Methods for increasing comb alias rejections are primarily based on comb zero rotation. Additional methods based on cosine filters, Chebyshev polynomials, cyclotomic polynomials, and cascade of combs with different decimation factors, among others, are also used in practice.

## REVIEW OF THE METHODS BASED ON COMB ZERO ROTATION

### Zero Rotation

The comb zero rotation was introduced by (Presti, 2000). The method consists of applying a clock-

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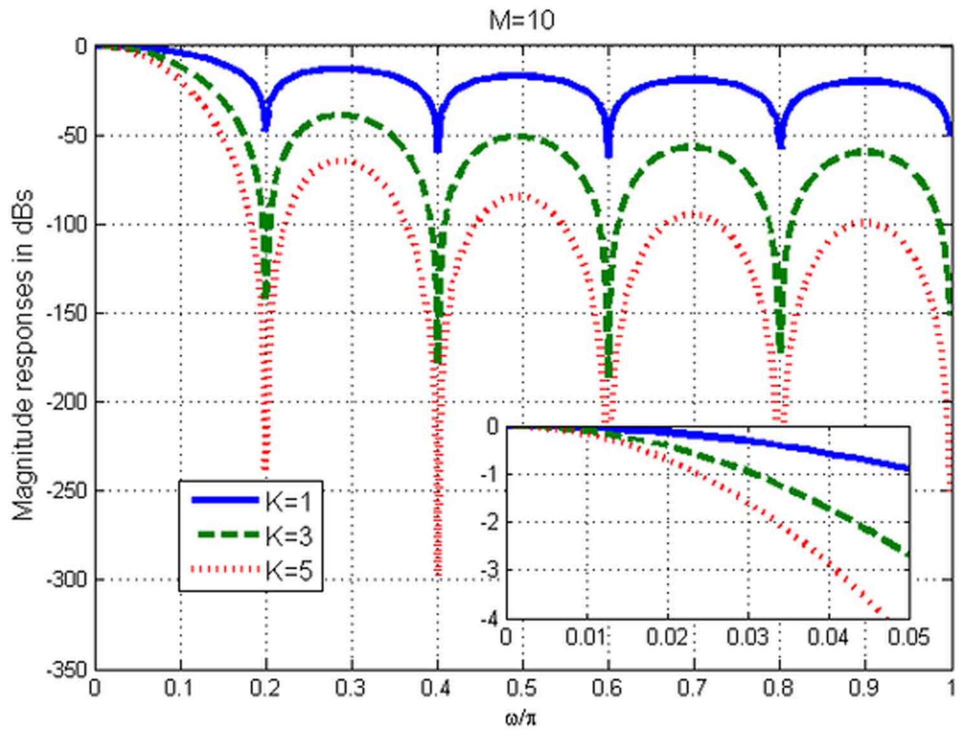
*Figure 1. The overall comb magnitude characteristics and the pass band zooms for M=10 and different filter orders*




wise rotation of $\beta$ radians to any zero of comb filter, thus obtaining the following transfer function:

$$H_u(z) = \frac{1}{M}\frac{1-z^{-M}e^{j\beta M}}{1-z^{-1}e^{j\beta}}. \tag{3}$$

By applying the rotation in the reverse direction, the following transfer function is obtained:

$$H_v(z) = \frac{1}{M}\frac{1-z^{-M}e^{-j\beta M}}{1-z^{-1}e^{-j\beta}}. \tag{4}$$

The filters (3) and (4) both have complex coefficients. However, their cascade has real coefficients:

$$\begin{aligned} H_r(z) &= H_u(z)H_v(z) \\ &= \frac{1}{M}\frac{1-z^{-M}e^{j\beta M}}{1-z^{-1}e^{j\beta}}\frac{1}{M}\frac{1-z^{-M}e^{-j\beta M}}{1-z^{-1}e^{-j\beta}}. \\ &= \frac{1}{M^2}\frac{1-2\cos(\beta M)z^{-M}+z^{-2M}}{1-2\cos(\beta)z^{-1}+z^{-2}} \end{aligned} \tag{5}$$

Figure 2(a) shows the zeros of a comb filter with $M$=5 and $K$=1. Similarly, Figure 2(b) shows the rotated zeros from (5), taking $\beta$=0.03. Note that the zeros of (5) are rotated around the original comb zero positions.

The cascade of the filter (5) and the comb filter is called rotated sinc (RS) filter:

$$\begin{aligned} H_R(z) &= H(z)H_r(z) \\ &= \frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}\frac{1}{M^2}\frac{1-2\cos(\beta M)z^{-M}+z^{-2M}}{1-2\cos(\beta)z^{-1}+z^{-2}}. \end{aligned} \tag{6}$$

Figure 3 compares the magnitude responses of a comb filter with parameters $M$=5, $K$=3 and the corresponding RS filter. The first folding band zooms are also shown.

It can be observed that due to the rotated zeros, the RS filter provides wider folding bands and consequently has better aliasing rejection than the corresponding comb filter and without additional deterioration in the pass band region. The method

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