Application of Soft Set in Game Theory

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INTRODUCTION

Soft set theory is a new mathematical approach to handle uncertainty based problems. It was introduced in (Molodtsov, 1999). Later on, fuzzy soft sets is defined in (Maji et al. 2001) and provided an application of soft set theory in decision-making problems (Maji et al., 2002). There are several theories which deal with vagueness and ambiguity. Some of these are: probability theory, interval mathematics and fuzzy set theory. All these models have their own drawbacks; such as fuzzy sets (Zadeh, 1965) are completely dependent on membership functions. There is no unique formula to define membership functions. Molodtsov observed that the main reason of these drawbacks is perhaps due to the inadequacy of parameterization tools. That led him to introduce soft set theory, which is a parameterized collection of subsets.

Game theory is defined as the mathematical model of interaction between rational, intelligent decision makers. In this chapter, we mention how game theory and soft sets are related. When we handle real life situations, we can observe that most of the information available is ambiguous or uncertain. Now a days, in most of the games we need to create human models or human behaviour. There exist a lot of approaches to describe human behavior in game theory. Some of them are pay function, choice functions etc. Molodtsov has introduced s-function (soft function) which keeps all good sides of choice function and eliminates the drawbacks of pay function and choice functions (Molodtsov, 1999). Deli and Cagman has studied further to associate soft set theory in game theory (Deli et al., 2013).

This chapter is further organized into ten sections. The next section contains definitions and notions of soft set theory. In the section three we have given some descriptions about the game theory. In the section four we have provided classification of games. Section five illustrates the representation of games in some subsections subsequently. In section six the definitions and notions related to both game theory and soft set theory are discussed. Section seven provides the definitions and notions about soft sets in decision making. In the section eight we provided the soft sets in decision making based on game theory which further describes about two-person soft game and n-person soft game under the subsections subsequently. Section nine provides future scope of this research work. Finally provide conclusions drawn from our research work. The chapter ends with a bibliography of sources referred for the compilation of our work.

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BACKGROUND

Soft set is a parameterized family of subsets defined over a universe associated with a set of parameters.

The definition of soft set is given below.

Definition 1(Molodtsov, 1999): A pair *(F, E)* is called a soft set over *U* iff *F* is a mapping of *E* into the set of all subsets of the universal set *U; i.e.*

$$F: E \rightarrow P(U) \quad (1)$$

where U is the universal set, E is the parameter set and *P(U)* is the power set *U*.

In other words, a soft set over *U* is a parameterised family of subsets of the universe *U*. For $e \in E$, *F(e)* can be called as the set of e-approximate elements of the soft set *(F, E).* So, a soft set can be represented as a collection of approximations. The parameter part of the approximation is called as predicate and for each parameter in *E* and the set containing all the elements of F*(e)* is called the value set of *e* in *(F, E).*

The pair *(U, E)* is often regarded as a soft universe. A parameter can be anything adverbial for the elements, such as a number, word, phrase or a sentence which can describe the value set more appropriately.

The way of describing an object in soft set theory differs principally from the way an object is described in classical mathematics. Normally, in classical mathematics a mathematical model of an object is constructed to define the notion of the exact solution of that model. Sometimes the mathematical model becomes too complicated to find the exact solution. So, we need the notion of approximate solution to get rid of these types of problems. However, in soft set theory, we are getting the solution to the problems by the opposite approach. The description of the object will have an approximate nature initially and so that we do not need to use the notion of exact solution.

There are no restrictions on the approximate description of the objects in soft set theory, which makes this model very convenient and can be easily applied in real life problems. We can use any type of parameterisation depending upon the preferences with the help of words and sentences, mappings, functions, real numbers, and so on.

Definition 2 (Maji et al., 2001): The pair *(F, E)* can be called as a fuzzy soft set over *U*, where *F* is a mapping given by

$$F: A \rightarrow I^U \quad (2)$$

where *U* is the initial universal set, *E* is the set of parameters, I^U is the power set of all fuzzy subsets of *U* and $A \subseteq E$.

For any element of *E*, the possible number of degrees of belongingness are real numbers lying in the interval of [0, 1] and hence infinite.

GAME THEORY

Game theory was introduced in (Von Neumann, 1944). Neumann is also known as the father of game theory. Game theory is a type of decision theory in which one's choice of action is determined after taking into account all possible alternatives available to an opponent playing the same game, rather than just by the possibilities of several outcomes.

In this section we discuss about strategic form games, the representation of games and the classification of games. Strategic form games are also called normal game forms.

Definition 3 (Strategic Form Games): A strategic form game ' Γ ' is a 3-tuple $\langle N, S_i, u_i \rangle$ where $i \in N$, N is the finite set of players, i.e., N= {1, 2, …, n}. S_i's are the strategy sets of players and