Chapter 11

Fuzzy Finite Element Method in Diffusion Problems

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ABSTRACT

Diffusion is an important phenomenon in various fields of science and engineering. It may arise in a variety of problems viz. in heat transfer, fluid flow problem and atomic reactors etc. As such these diffusion equations are being solved throughout the globe by various methods. It has been seen from literature that researchers have investigated these problems when the material properties, geometry (domain) etc. are in crisp (exact) form which is easier to solve. But in real practice the parameters used in the modelled physical problems are not crisp because of the experimental error, mechanical defect, measurement error etc. In that case the problem has to be defined with uncertain parameters and this makes the problem complex. In this chapter related uncertain differential equation of various diffusion problems will be investigated using finite element method, which may be called fuzzy or interval finite element method.

1. INTRODUCTION

Uncertainty plays a vital role in various fields of engineering and science. These uncertainties occur due to incomplete data, impreciseness, vagueness, experimental error and different operating conditions influenced by the system. Different authors proposed various methods to handle uncertainty. They have used some probabilistic or statistical method as a tool to operate uncertain parameters. Using these probabilistic and statistical methods authors have calculated variability of involved parameters through the heat and mass transfer model. In this regard, the Monte Carlo method is used to solve heat and mass propagation problem. It essentially involves a large number of process samples which are obtained by numerically solving the problem for artificially generated random parameter samples. Monte Carlo method have been used to analyse thermal food processes with variable parameters (Wang et al., 1991; Varga et al., 2000; Caro-Corrales et al., 2002; Demir et al., 2003; Halder et al., 2007; Laguerre and Flick,

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2010). Deng and Liu (2002) implemented Monte Carlo method to solve the direct bio heat transfer problems. They have demonstrated the bio heat transfer problem with transient or space-dependent boundary conditions, blood perfusion, metabolic rate, and volumetric heat source for tissue. It may be noted that in this process we need more number of observed data or experimental results to analyse the problems. Practically it may not be possible to get a large number of data because it requires more number of experiments to perform. So instead of probabilistic or statistical method we may use interval or fuzzy parameters to handle uncertainty which may require less number of data. In general traditional interval/fuzzy arithmetic are complicated to manage the problem rigorously. Accordingly, we have proposed here new techniques to handle such difficulty which are simple and efficient. Here the traditional interval arithmetic is modified for the said problem and a simpler method is proposed to compute interval arithmetic. The idea of modified interval arithmetic is then extended for uncertain fuzzy numbers also. As such uncertain parameters are taken as fuzzy. Then the fuzzy numbers are converted into interval using α-cut techniques. This fuzzy numbers contain left monotonically increasing and right monotonically decreasing functions respectively. Here two types of fuzzy numbers viz. Triangular Fuzzy Number (TFN) and Trapezoidal Fuzzy Number (TRFN) have been considered for the investigation. Above technique is used to study the quantification of uncertain distribution of temperatures for different types of heat conduction problems. In this context for the sake of completeness we may mention that conduction of heat means transfer of heat energy within the body due to the temperature gradient. Heat spontaneously flows from a body having higher temperature to lower temperature. But in absence of external driving fluxes it approaches to thermal equilibrium. There are two types of conduction such as steady and unsteady state. Steady state conduction is a form of conduction where the temperature differences deriving by the conduction remains constant and it is independent of time.

As discussed above the uncertain heat conduction problems may be solved by using the sense of fuzziness along with the numerical techniques. The numerical techniques viz. Finite Difference Method (FDM), Finite Element Method (FEM), Finite Volume Method (FVM) are used to study the solution of the mentioned problems.

Magnus et al. (2011) used finite difference method in his paper to model and solve the governing ground water flow rates, flow direction and hydraulic heads through an aquifer. Muhieddine et al. (2009) described one dimensional phase change problem. They have used vertex centred finite volume method to solve the problem.

Edward and Robert (1966) used FEM to solve heat conduction problem and analyse it. A non-iterative, finite element-based inverse method for estimating surface heat flux histories on thermally conducting bodies is presented by Ling et al. (2003). They considered both linear and non-linear problems, and sequentially minimizes the least square error norm between corresponding sets of measured and computed temperatures. Further Onate et al. (2006) used Galerkin FEM for convective–diffusive problems with sharp gradients using finite calculus. In view of the above literatures, it reveals that the traditional finite element method may easily be used where the parameters or the values are exact, that is in crisp form. But in actual practice the values may be in a region of possibility or we can say the values are uncertain. These uncertain parameters give uncertain model predictions. Although the uncertainty may be reduced by appropriate experiments but still it may also give the variability in the parameters. Then finite element perturbation method is used by Nicolaï and De Baerdemaeker (1993) and Nicolaï et al. (2000) for heat conduction problem considering uncertain physical parameters. Further Nicolaï et al. (1999a and 1999b) found the temperature in heat conduction problem for randomly varying parameters

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