# Digital Filters

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## INTRODUCTION

A *signal* is defined as any physical quantity that varies with changes of one or more independent variables, and each can be any physical value, such as time, distance, position, temperature, or pressure (Elali, 2003; Smith, 2002). The independent variable is usually referred to as "time". Examples of signals that we frequently encounter are speech, music, picture, and video signals. If the independent variable is continuous, the signal is called *continuous-time signal* or *analog signal*, and is mathematically denoted as $x(t)$. For *discrete-time signals*, the independent variable is a discrete variable; therefore, a discrete-time signal is defined as a function of an independent variable $n$, where $n$ is an integer. Consequently, $x(n)$ represents a sequence of values, some of which can be zeros, for each value of integer $n$. The discrete–time signal is not defined at instants between integers, and it is incorrect to say that $x(n)$ is zero at times between integers. The amplitude of both the continuous and discrete-time signals may be continuous or discrete. *Digital signals* are discrete-time signals for which the amplitude is discrete. Figure 1 illustrates the analog and the discrete-time signals.

Most signals that we encounter are generated by natural means. However, a signal can also be generated synthetically or by computer simulation (Mitra, 2006).

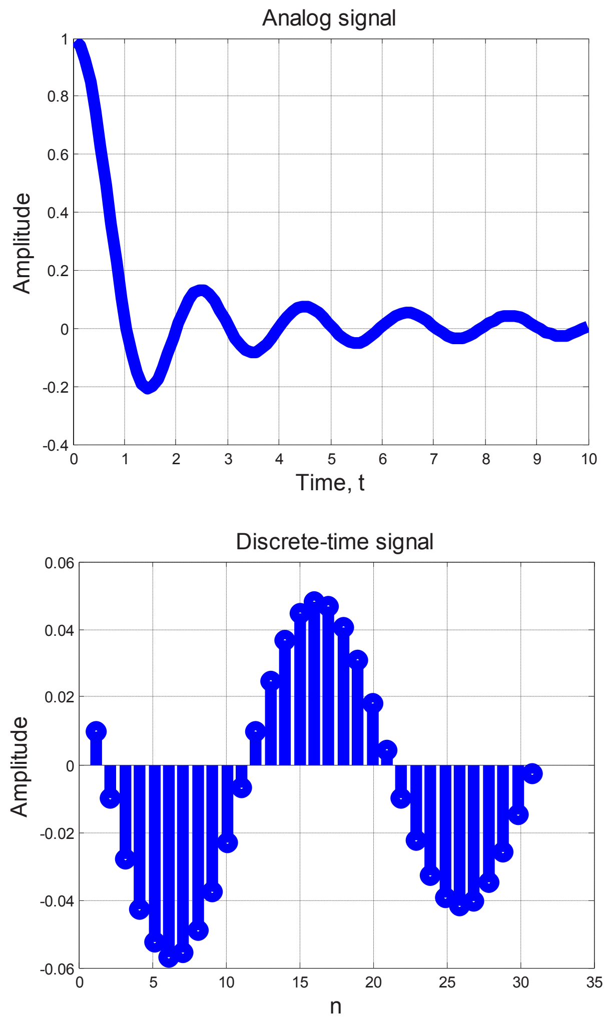
Signal carries information, and the objective of *signal processing* is to extract useful information carried by the signal. The method of information extraction depends on the type of signal and the nature of the information being carried by the signal. "Thus, roughly speaking, signal processing is concerned with the mathematical representation of the signal and algorithmic operation carried out on it to extract the information present," (Mitra, 2006, pp. 1).

*Analog signal processing* (ASP) works with the analog signals, while *digital signal processing* (DSP) works with digital signals. Since most of the signals that we encounter in nature are analog, DSP consists of these three steps:

- A/D conversion (transformation of the analog signal into the digital form);
- Processing of the digital version; and
- Conversion of the processed digital signal back into an analog form (D/A).

We now mention some of the advantages of DSP over ASP (Diniz, Silva, & Netto, 2002; Ifeachor & Jervis, 2001; Mitra, 2006; Stearns, 2002; Stein, 2000):

*Figure 1. Examples of analog and discrete-time signals*



- Less sensitivity to tolerances of component values and independence of temperature, aging, and many other parameters;
- Programmability, that is, the possibility to design one hardware configuration that can be programmed to perform a very wide variety of signal processing tasks simply by loading in different software;
- Several valuable signal processing techniques that cannot be performed by analog systems, such as for example linear phase filters;
- More efficient data compression (maximum amount of information transferred in the minimum amount of time);
- Any desirable accuracy can be achieved by simply increasing the word length;
- Applicability of digital processing to very low frequency signals, such as those occurring in seismic applications (An analog processor would be physically very large in size.); and
- Recent advances in very large scale integrated (VLSI) circuits make it possible to integrate highly-sophisticated and complex digital signal processing systems on a single chip.

Nonetheless, DSP has some disadvantages (Diniz, Silva, & Netto, 2002; Ifeachor & Jervis, 2001; Mitra, 2006; Stein, 2000):

- **Increased complexity:** The need for additional pre-and post-processing devices such as A/D and D/A converters and their associated filters and complex digital circuitry;
- The limited range of frequencies available for processing; and
- **Consummation of power:** Digital systems are constructed using active devices that consume electrical power, whereas a variety of analog processing algorithms can be implemented using passive circuits employing inductors, capacitors, and resistors that do not need power.

In various applications, the aforementioned advantages by far outweigh the disadvantages and, with the continuing decrease in the cost of digital processor hardware, the field of digital signal processing is developing fast. "Digital signal processing is extremely useful in many areas, like image processing, multimedia systems, communication systems, audio signal processing" (Diniz, Silva, & Netto, 2002, pp. 2-3).

*Figure 2. Digital filter*


The system which performs digital signal processing, that is, transforms an input sequence $x(n)$ into a desired output sequence $y(n)$, is called a *digital filter* (see Figure 2).

We consider a filter to be a *linear-time invariant system* (LTI). The linearity means that the output of a scaled sum of the inputs is the scaled sum of the corresponding outputs, known as the principle of superposition. The time invariance says that a delay of the input signal results in the same delay of the output signal.

## TIME-DOMAIN DESCRIPTION

If the input sequence $x(n)$ is a unit impulse sequence $\delta(n)$,

$$\delta(n) = \begin{cases} 1 & for \quad n = 0 \\ 0 & otherwise \end{cases}, \qquad (1)$$

then the output signal represents the characteristics of the filter called the *impulse response*, and denoted by $h(n)$. We can, therefore, describe any digital filter by its impulse response $h(n)$.

Depending on the length of the impulse response $h(n)$, digital filters are divided into filters with the *finite impulse response* (FIR) and *infinite impulse response* (IIR).

In practical applications, one is only interested in designing stable digital filters, that is, whose outputs do not become infinite. The stability of a digital filter can be expressed in terms of the absolute values of its unit sample responses (Mitra, 2006; Smith, 2002),

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty. \qquad (2)$$

Because the summation (2) for an FIR filter is always finite, FIR filters are always stable. Therefore, the stability problem is relevant in designing IIR filters.

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