Chapter 11
On Bipolar Fuzzy B-Subalgebras of B-Algebras

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ABSTRACT

Based on the concept of bipolar fuzzy set, a theoretical approach of B-subalgebras of B-algebras are established. Some characterizations of bipolar fuzzy B-subalgebras of B-algebras are given. We have shown that the intersection of two bipolar fuzzy B-subalgebras is also a bipolar fuzzy B-subalgebra, but for the union it is not always true. We have also shown that if every bipolar fuzzy B-subalgebras has the finite image, then every descending chain of B-subalgebras terminates at finite step.

INTRODUCTION

After the introduction of fuzzy set [24], many new approaches and theories treating imprecision and uncertainty have been proposed, such as the generalized theory of uncertainty (GTU) introduced by Zadeh (1965) and the intuitionistic fuzzy sets introduced by Atanassov (1999) and so on. Among these theories, a well-known extension of the classic fuzzy set is bipolar fuzzy set theory, which was pioneered by Zhang (1994). Since then, many researchers have investigated this topic and obtained some meaningful conclusions.

In traditional fuzzy sets the membership degree range is [0,1]. The membership degree is the degree of belonging ness of an element to a set. The membership degree 1 indicates that an element completely belongs to its corresponding set, the membership degree 0 indicates that an element does not belong to the corresponding set and the membership degree on the interval (0,1) indicate the partial membership to the corresponding set. Sometimes, membership degree also means the satisfaction degree of elements to some property corresponding to a set and its counter property. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is increased from the interval [0,1] to the interval [–1,1]. In a bipolar fuzzy set the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degrees on (0,1] indicate that elements somewhat satisfy the property and the membership degrees on [–1,1] indicate that elements somewhat satisfy the implicit counter-property.

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Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are essentially different sets [12]. In many domains, it is important to be able to deal with bipolar information. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible.

The study of *BCK*/*BCI*-algebras was initiated by Imai and Iseki in 1966 as a generalization of the concept of set-theoretic difference and propositional calculus. Neggers and Kim (2001, 2002) introduced a new notion, called a *B*-algebras which is related to several classes of algebras of interest such as *BCK*/*BCI* -algebras. Jun et al. (2002, 2009) applied the concept of fuzzy sets to *B*-algebras. Ahn and Bang [1] discussed some results on fuzzy subalgebras in *B*-algebras. Saeid (2006) introduced fuzzy topological *B*-algebras. Senapati et al. (2012, 2013) introduced fuzzy and interval-valued fuzzy closed ideals of *B*-algebras. They investigated fuzzy *B*-subalgebras with respect *t*-norm. They also give relationship between fuzzy dot subalgebras and fuzzy dot ideals of *B*-algebras.

Kim and Kim (2008) introduced the notion of *BG*-algebras, which is a generalization of *B*-algebras. Senapati et al. (2012, 2013, 2014) done lot of works on *BG* -algebras.

Lee (2000, 2004, 2009) introduced the notion of bipolar fuzzy subalgebras and ideals in *BCK*/*BCI* -algebras. The concept of bipolar valued fuzzy translation and bipolar valued fuzzy *S*-extension of a bipolar valued fuzzy subalgebra in *BCK*/*BCI* -algebra was introduced by Jun et al. (2008). Motivated by this, in this paper, the notions of bipolar fuzzy *B* -subalgebras of *B* -algebras are introduced and their properties are investigated.

PRELIMINARIES

We first recall some basic concepts which are used to present the paper.

Definition 2.1 *(B* -algebra) A non-empty set *X* with a constant 0 and a binary operation * is said to be *B* -algebra if it satisfies the following axioms

1. x *x = 0
2. x * 0 = x
3. (x * y) * z = x * (z * (0 * y)), for all x, y, z ∈ X.

A non-empty subset *S* of a *B*-algebra *X* is called a *B*-subalgebra [13] of *X* if $x * y \in S$, for all $x, y \in S$. We can define a partial ordering "≤" by *x*≤*y* if and only if *x***y*=0.

Definition 2.2: *(B* -subalgebra) A non-empty subset S of a *B* -algebra *X* is called a *B* -subalg ebra of *X* if $x * y \in S$, for all $x, y \in S$.

From this definition it is observed that, if a subset *S* of a *B* -algebra satisfies only the closer property, then *S* becomes a *B* -subalgebra.

Definition 2.3: *(Fuzzy set) Let X* be the collection of objects denoted generally by *x*, then a fuzzy set *A* in *X* is defined as $A = \{\langle x, \mu_A(x) \rangle : x \in X\}$, where $\mu_A(x)$ is called the membership value of *x* in *A* and $0 \le \mu_A(x) \le 1$.

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