Chapter 4
Solution of Basic Inventory Model in Fuzzy and Interval Environments:
Fuzzy and Interval Differential Equation Approach

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ABSTRACT

In this present paper a basic inventory model is solved in different imprecise environments. Four different cases are discussed: 1) Crisp inventory model, that is, the quantity at present and demand is crisp number; 2) Inventory model in fuzzy environment, that is, the quantity and demand both are fuzzy number; 3) Inventory model in interval environment, that is, the quantity and demand both are interval number and lastly; 4) Inventory model in time dependent fuzzy environment, that is, quantity and demand are both time dependent fuzzy number. Different numerical examples are used to illustrate the model as well as to compute the efficiency of imprecise differential equation approach to solve the model.

INTRODUCTION

The classical economic order quantity (EOQ) model was first developed by Harris (1915). Lots of researcher works on inventory control system and gives valuable contribution (e.g., Chang & Dye, 1999; Donaldson, 1977). A real-life inventory control model cannot be modeled without uncertainty or impreciseness in the parameters and/or variables. This uncertainty or impreciseness may be defined in stochastic and non-stochastic (fuzzy) sense and attempts to formulate and analyze such models (e.g., Chiang et al., 2005; Lee et al., 1999).

Though the demand and initial quantity of an item depends on several factors (such as selling rice, marketing cost, display of goods in showroom etc.), thus in reality it was estimated as random parameters with a probability distribution if sufficient past data available. These uncertain quantities can also be

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estimated as fuzzy or interval parameter if past data is insufficient. We can assume that the sufficient past data are not available. Thus it is better to estimate parameters of quantity and demand as fuzzy or interval number rather than crisp or random number.

In this chapter, we look at the inventory problem with first order fuzzy differential equation (FDE) and interval differential equation (IDE). We solve the inventory model using FDE and IDE concepts. Presence of fuzzy and interval demand and quantity the model leads to FDE and IDE. Now the question arise in the readers mind that, "what is new approach for solving fuzzy and interval inventory model?". In the last few decay's many researcher consider the inventory models in fuzzy environment. But there is a problem. They first solve the inventory model with crisp number and at the solution then they substitute the concerned parameter by fuzzy number. Here the concept of fuzzy differential equation is missing. Moreover the application of fuzzy concept is violated (e.g., Guchhait et al. () etc). We consider two cases where the inventory model is considered with fuzzy differential equation. The solution procedure is used here namely generalized Hukuhara derivative approach, which is more recent general concept for solving fuzzy differential equation.

The uses of the interval number on this inventory model is also illustrates in a case. The interval number is taken as a new orientation. In interval environment the differential equation in present model converted to a interval differential equation. And by using the concepts of interval differential equation we solve the problem.

The concept of the fuzzy derivative was first initiated by Chang and Zadeh (1972). It was followed up by Dubois and Prade (1982). Other methods have been smeared by Puri and Ralescu (1983) and Goetschel and Voxman (1986). The concept of differential equations in a fuzzy environment was first formulated by Kaleva (1987). In fuzzy differential equation all derivative is deliberated as either Hukuhara or generalized derivatives. The Hukuhara differentiability has a deficiency (see Bede & Gal, 2005). The solution turns fuzzier as time goes by. Bede (2006) exhibited that a large class of BVPs has no solution if the Hukuhara derivative is applied. To exceeds this difficulty, the concept of a generalized derivative was developed and fuzzy differential equations were smeared using this concept (see Bencsik et al. (2007)). Khastan and Nieto (2010) set up the solutions for a large enough class of boundary value problems using the generalized derivative. Obviously the disadvantage of strongly generalized differentiability of a function in comparison H-differentiability is that, a fuzzy differential equation has no unique solution (see Bede and Gal (2005)). Recently, Stefanini (2008) by the concept of generalization of the Hukuhara difference for compact convex set, introduced generalized Hukuhara differentiability (see Stefanini and Bede (2009)) for fuzzy valued function and they displayed that, this concept of differentiability have relationships with weakly generalized differentiability and strongly generalized differentiability.

There are many approaches for solving FDE. Some researchers transform the FDE into equivalent fuzzy integral equation and then solve this (see Allahviranloo et al. (2011)). Another one is Zadeh extension (1975) principle method. In this method first solve the associated ODE and lastly fuzzify the solution and check whether it is satisfied or not. For details see Buckley and Feuring (2000, 2001). In the third approach, the fuzzy problem is converted to a crisp problem. Hüllermeier (1997), uses the concept of differential inclusion. In this way, by taking an α-cut of the initial value and the solution, the given differential equation is converted to a differential inclusion and the solution is accepted as the α-cut of the fuzzy solution. Laplace transform method is use many where in linear FDE (see Allahviranloo and Ahmadi, 2010). Recently, Mondal and Roy (2013) solve the first order Linear FDE by Lagrange multiplier method. Using generalized Hukuhara differentiability concept we transform the given FDE into two ODEs. And this ODEs also a differential equation involving the parametric form of a fuzzy number.