Chapter 17
Solution of Some Differential Equation in Fuzzy Environment by Extension Principle Method and Its Application in Biomathematics

Sankar Prasad Mondal
National Institute of Technology, India

ABSTRACT

The concept of fuzzy differential equations is very important for new developments of model in various fields of science and engineering problems in uncertain environments because this theory represent a natural way to modeling dynamical system under uncertain environment. In this way we can modeled mathematical biology problem associated with differential equation in fuzzy environment and solved them. In this chapter we solve two mathematical biology models which are taken in fuzzy environment. A one species prey predator model is considered with fuzzy initial data. Whereas an insect population model are described with fuzzy initial value. The solution procedures of the fuzzy differential equation are taken as extension principle method.

1. INTRODUCTION

1.1 Fuzzy Differential Equation

The topic "fuzzy differential equation" (FDE) has been speedily developing in recent years. The appliance of fuzzy differential equations is an inherent way to model dynamic systems under possibilistic uncertainty (Zadeh (2005)). The concept of the fuzzy derivative was first initiated by Chang and Zadeh (1972). It was followed up by Dubois and Prade (1982). Other methods have been smeared by Puri and Ralescu (1983) and Goetschel and Voxman (1986). The concept of differential equations in a fuzzy envi-

ronment was first formulated by Kaleva (1987). In fuzzy differential equation all derivative is deliberated as either Hukuhara or generalized derivatives. The Hukuhara differentiability has a deficiency (Bede et al., 2005; Diamond et al., 1994). The solution turns fuzzier as time goes by. Bede (2006) exhibited that a large class of BVPs has no solution if the Hukuhara derivative is applied. To exceeds this difficulty, the concept of a generalized derivative was developed (Bede & Gal, 2005; Cano & Flores, 2008) and fuzzy differential equations were smeared using this concept (Bede & Rudas, 2007; Cano et al., 2007; Cano et al., 2008; Stefanini & Bede, 2009). Khastan and Nieto (2010) set up the solutions for a large enough class of boundary value problems using the generalized derivative. Obviously the disadvantage of strongly generalized differentiability of a function in comparison H-differentiability is that, a fuzzy differential equation has no unique solution (Bede & Gal, 2005). Recently, Stefanini and Bede (2008) by the concept of generalization of the Hukuhara difference for compact convex set, introduced generalized Hukuhara differentiability (Stefanini & Bede, 2009) for fuzzy valued function and they displayed that, this concept of differentiability has relationships with weakly generalized differentiability and strongly generalized differentiability.

There are many approaches for solving FDE. Some researchers transform the FDE into equivalent fuzzy integral equation and then solve this (Allahviranloo et al., 2011; Chen et al., 2008; Regan et al., 2003). Another one is Zadeh extension principle method. In this method first solve the associated ODE and lastly fuzzify the solution and check whether it is satisfied or not. For details see Buckley and Feuring (2000, 2001). In the third approach, the fuzzy problem is converted to a crisp problem. Hüllermeier (1997), uses the concept of differential inclusion. In this way, by taking an α-cut of the initial value and the solution, the given differential equation is converted to a differential inclusion and the solution is accepted as the α-cut of the fuzzy solution. Laplace transform method is use many where in linear FDE (Allahviranloo & Ahmadi, 2010; Tolouti & Ahmadi, 2010). Recently, Mondal and Roy (2013) solve the first order Linear FDE by Lagrange multiplier method. Using generalized Hukuhara differentiability concept we transform the given FDE into two ODEs. And this ODEs also a differential equation involving the parametric form of a fuzzy number.

1.2 Work Done on Biomathematics Problem in Fuzzy Environment

In Barros et al. (2000) take fuzzy population dynamics model with fuzzy initial value and solve the problem. Akin and Oruc (2012) consider a prey predator model with fuzzy initial value. Here they solve the equation by generalized derivative of fuzzy function concept. Zarei et al. (2012) give a formulation on fuzzy HIV modeling. Diniz et al. (2001) consider a fuzzy Cauchy problem and apply in the behavior of decay of the biochemical oxygen demand in water. Nounou et al. (2012) discuss fuzzy intervention in biological phenomena. Fuzzy delay predator-prey system and their existence theorem and oscillation property of solution is nicely delivered by Barzinji (2014). Symptomatic HIV virus infected population with fuzzy concept is done by Jafelice et al (2004). The interaction of predator prey with uncertain initial population sizes was considered in Omar et al. (2011). Predator–prey fuzzy model is solved by Peixoto (2008) by fuzzy rule base method. Ahmad et al. (2012) solve a biological population model by numerical method, Euler's method. Here system of fuzzy differential equation is solved. Zaini and Baets (2009), solve the predator-prey model with fuzzy initial populations by Runge-Kutta method. Optimal control of HIV infection by using fuzzy dynamical systems is discussed by Najariyan et al. (2011). Mann et al. (2013) use delay differential equation in predator-prey interaction and analysis on the stability of steady state. Pal and Mahapatra (2014) solve a bio-economic modeling of two-prey and one-predator fishery

26 more pages are available in the full version of this document, which may be purchased using the "Add to Cart" button on the publisher's webpage: www.igi-global.com/chapter/solution-of-some-differential-equation-in-fuzzy-environment-by-extension-principle-method-and-its-application-in-biomathematics/157497

Related Content

Incremental Load in a Data Warehousing Environment
Nayem Rahman (2012). *Insights into Advancements in Intelligent Information Technologies: Discoveries (pp. 161-177).*
www.irma-international.org/chapter/incremental-load-data-warehousing-environment/64375

Time and Space Reasoning for Ambient Systems
Radja Radja Boukharrou, Jean-Michel Iliéand Djamel Eddine Saidouni (2017). *International Journal of Ambient Computing and Intelligence (pp. 38-57).*
www.irma-international.org/article/time-and-space-reasoning-for-ambient-systems/183619

Mastery of Learning via Social Networking Technology: A Comprehensive Literature Review With a Systems Approach
Seda Khadimally (2022). *Applications of Machine Learning and Artificial Intelligence in Education (pp. 142-161).*
www.irma-international.org/chapter/mastery-of-learning-via-social-networking-technology/299225

Existence of Fuzzy Zhou Bargaining Sets in TU Fuzzy Games
Wenbo Yang, Jiuqiang Liuand Xiaodong Liu (2018). *International Journal of Fuzzy System Applications (pp. 46-55).*
www.irma-international.org/article/existence-of-fuzzy-zhou-bargaining-sets-in-tu-fuzzy-games/195676

Digitization Projects for Cultural Heritage Materials: A Study With Special Reference to Arabic, Persian, and Urdu Manuscripts
Amreen Tajand Bhakti Gala (2024). *AI-Assisted Library Reconstruction (pp. 238-255).*
www.irma-international.org/chapter/digitization-projects-for-cultural-heritage-materials/343589