Chapter 21
Verification of Iterative Methods for the Linear Complementarity Problem:
Verification of Iterative Methods for LCPs

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ABSTRACT

In the present chapter, we give an overview of iterative methods for linear complementarity problems (abbreviated as LCPs). We also introduce these iterative methods for the problems based on fixed-point principle. Next, we present some new properties of preconditioned iterative methods for solving the LCPs. Convergence results of the sequence generated by these methods and also the comparison analysis between classic Gauss-Seidel method and preconditioned Gauss-Seidel (PGS) method for LCPs are established under certain conditions. Finally, the efficiency of these methods is demonstrated by numerical experiments. These results show that the mentioned models are effective in actual implementation and competitive with each other.

INTRODUCTION

For a given real vector $q \in R^n$ and a given matrix $A \in R^{n\times n}$ the linear complementarity problem abbreviated as *LCP (A, q)*, consists in finding vectors $z \in R^n$ such that

$$\begin{cases} w = Az + q \\ z \geq 0, w \geq 0 \\ z^T w = 0 \end{cases} \tag{1.1}$$

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where, z^T denotes the transpose of the vector z. Note that the components of a solution (w, z) are *complementary* in the sense that if $w_i > 0$, then $z_i = 0$, and if $z_i > 0$, then $w_i = 0$. It may be the case that $w_i = z_i = 0$ however.

The Linear Complementarity Problems (*LCPs*) is one of the fundamental problems in optimization and mathematical programming (Murty & Yu 1988; Cottle, Pang et al., 1992). The Eq.(1.1) is a general problem which unifies bounteous practical problems in various scientific computing, economics and engineering areas. For example, linear and quadratic programming (Murty & Yu, 1988), economics and Nash equilibrium point of a bi-matrix game (Gale 1960; Lemke & Howson, 1964; Lemke, 1965), invariant capital stock (Cottle, Pang et al., 1992), optimal stopping in Markov chains(Cottle, Pang et al., 1992), contact and structural mechanics (Pfeiffer & Glocker, 2000), free boundary problem for journal bearings (Lin & Cryer 1985), traffic equilibriums(Ferris & Pang, 1997), circuit simulation (Van Eijndhoven, 1986), geometry (Du Val, 1940), etc. Therefore, it deserves to pay much more attention to the researches on both theoretical properties and numerical methods about the solution of this problem. So, many direct and iterative methods have been developed for its solution; see (Murty & Yu, 1988; Cottle, Pang et al, 1992) and the references therein.

The early motivation for studying the LCPs was because the KKT optimality conditions for linear and quadratic programs constitute an LCP of the form Eq.(1.1). However, the study of LCPs really came into prominence since the 1960s and in several decades, many methods for solving the LCP (A, q) have been introduced. Most of these methods originate from those for the system of the linear equations where may be classified into two principal kinds. The book by Cottle et al. (Cottle, Pang et al., 1992) is a good reference for direct methods developed to solve LCP. Another important class of methods used to tackle LCPs is the iterative methods. A more general iterative method, attributed to Christopherson (Christopherson, 1941), has been analyzed and clarified by Cryer (Cryer, 1971). In contrast to real attractiveness of iterative methods, lack of strength in these methods over direct methods and slow rate of convergence where unexpected, will make their real applications doubtful. The key of this problem is using preconditioning techniques. A preconditioner is defined as an auxiliary approximate solver which will be combined with an iterative method. According to critical importance of spectral radius, in preconditioning, we finding a more desired spectral radius. In other words, for the iterative solution of a linear system $Ax=b$, in order to accelerate the convergence of iterative solvers, preconditioning transforms the system to $PAx=Pb$, where P is a linear operator, called the preconditioner (Saberi Najafi & Edalatpanah, 2013, 2014) . Recently, Duan *et.al (2012)*, present a new preconditioned Gauss-Seidel iterative method for solving the linear complementarity problem, whose the certain elementary operations row are performed on system matrix A before applying the Gauss-Seidel iterative method. Moreover the sufficient conditions for guaranteeing the convergence of the new preconditioned Gauss-Seidel iterative method are presented. In this chapter, we study some models of iterative methods for linear complementarity problems (LCPs). Another main purpose in this chapter is further study on Duan *et.al* (2012) preconditioned method for LCP. To accomplish this purpose, we use these preconditioners and show that the preconditioned Gauss-Seidel (PGS) methods are superior to the basic Gauss-Seidel iterative method point of view rate of convergence and computing efficiency. Finally, numerical results are also given to illustrate the efficiency of these algorithms.