Chapter 2
Robust Two-Stage and Multistage Optimization: Complexity Issues and Applications

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ABSTRACT

This chapter is intended as an overview of robust optimization models related to optimization problems subject to uncertain data, with special focus on the case when uncertainty impacts the right-hand side coefficients in the constraints. Two-stage as well as multistage models are addressed, emphasizing links with applications and computational complexity issues. A class of multistage robust optimization problems for which exact optimal strategies can be efficiently computed (via a robust dynamic programming recursion) is discussed. An application to a multiperiod energy production planning problem is presented into detail, and computational results are reported.

1. INTRODUCTION

The problem of taking the best possible decisions to operate a complex system in an uncertain environment arises in a huge variety of applications in Engineering, Management Science, Operations Research. A rich body of literature related to the question of how to take uncertainty into account within optimization problems has been developed over the years providing a wide variety of models and algorithms, including scenario-based stochastic programming (e.g. Birge, 1985), stochastic dynamic programming (Ross, 1983), chance constraint programming (Charnes & Cooper, 1959).

A well-known limitation of these approaches is that they assume that the uncertain parameters of the problems addressed can be considered as random variables with *perfectly known probability distributions*; however such an assumption is often quite unrealistic because of the lack of input data or the absence of systematic prior statistical analysis.

In order to overcome the above-mentioned limitations, the field of *robust optimization* has emerged and developed in the last 15 years or so in order to handle situations in which probability distributions

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are essentially unknown, and when it is felt acceptable to optimize the system against the most adverse scenarios taken in a specified set of scenarios, called the *uncertainty set*; some basic references are Ben Tal and Nemirovski (1999, 2000, 2002), Ben Tal *et al.* (2004), Bertsimas and Sim (2003, 2004), Iyengar (2005)

The main focus of this chapter is on robust optimization models and solution algorithms, and it is intended to provide both a *review of already known results* concerning two-stage and multistage robust optimization models (sections 2 and 3), and *new results* concerning: a) comparison of robust optimization models and stochastic optimization models in their capability of taking risk concerns into account (section 2.2); b) a class of efficiently solvable special cases of multistage robust optimization models (section 4). Central to the definition of this class is the concept of *state-space representable uncertainty* set; it includes several types of uncertainty sets already proposed in the literature, and, as an original contribution to the present work, it is shown here to provide a compact representation of uncertainty sets related to Markov chains (section 4.4). Links with applications will also be emphasized throughout the chapter and computational results will be discussed (section 5) in connection with a multiperiod energy production planning problem.

2. BACKGROUND: ROBUST 2-STAGE (LINEAR) OPTIMIZATION PROBLEMS

In this section we fist introduce a generic model for the class of robust 2-stage (linear) optimization problems featuring right-hand side uncertainty (RHS-uncertainty). We next discuss links with related stochastic models and illustrate how the two approaches differ in their capability of taking risk concerns into account. Finally, we provide an overview of possible applications, emphasizing known complexity (NP-hardness) results and polynomial-time solvable special cases.

2.1 Robust 2-Stage Optimization with RHS Uncertainty

For the sake of simplicity in the presentation, we restrict here to the case of *linear* programs with uncertain right-hand side vectors. Of course extensions to nonlinear programming models are possible (an example of such extensions can be found e.g. in (Minoux, 2014)).

Also we note that problems involving nonlinear but *convex* constraints and/or objective function can be transformed (using tangential approximation), up to any prescribed level of accuracy, into LP problems; thus assuming linearity in the generic model considered here is less restrictive than it might appear at first sight.

The generic model of 2-stage robust LP problems discussed in the sequel is based on the deterministic LP problems in variables *x, y*:

$$\text{(I)} \qquad \begin{cases} \textit{Minimize } \gamma^T x + q^T y \\ \textit{subject to}: \\ Fx + Hy = b \\ \chi \in x, y \geq 0 \end{cases} \tag{2.1}$$