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Data Mining for Combining Forecasts in Inventory Management

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INTRODUCTION

The traditional approach to forecasting involves choosing the forecasting method judged most appropriate of the available methods and applying it to some specific situations. The choice of a method depends upon the characteristics of the series and the type of application. The rationale behind such an approach is the notion that a "best" method exists and can be identified. Further that the "best" method for the past will continue to be the best for the future. An alternative to the traditional approach is to aggregate information from different forecasting methods by aggregating forecasts. This eliminates the problem of having to select a single method and rely exclusively on its forecasts.

Considerable literature has accumulated over the years regarding the combination of forecasts. The primary conclusion of this line of research is that combining multiple forecasts leads to increased forecast accuracy. This has been the result whether the forecasts are judgmental or statistical, econometric or extrapolation. Furthermore, in many cases one can make dramatic performance improvements by simply averaging the forecasts.

BACKGROUND OF COMBINATION OF FORECASTS

The concept of combining forecasts started with the seminal work 35 years ago of Bates and Granger (1969). Given two individual forecasts of a time series, Bates and Granger (1969) demonstrated that a suitable linear combination of the two forecasts may result in a better forecast than the two original ones, in the sense of a smaller error variance. Table 1 shows an example in which two individual forecasts (1 and 2) and their arithmetic mean (combined forecast) were used to forecast 12 monthly data of a certain time series (actual data).

The forecast errors (i.e., actual value – forecast value) and the variances of errors are shown in Table 2.

From Table 2, it can be seen that the error variance of individual forecast 1, individual forecast 2, and the combined forecast are 196, 188 and 150, respectively. This shows that the error variance of the combined forecast is smaller than any one of the individual forecasts and hence demonstrates an example how combined forecast may work better than its constituent forecasts.

Bates and Granger (1969) also illustrated the theoretical base of combination of forecasts. Let X_{1t} and X_{2t} be two individual forecasts of Y_t at time t with errors:

Table 1. Individual and combined forecasts

Actual Data (Monthly Data)	Individual Forecast 1	Individual Forecast 2	Combined Forecast (Simple Average of Forecast 1 and Forecast 2)
196	195	199	197
196	190	206	198
236	218	212	215
235	217	213	215
229	226	238	232
243	260	265	262.5
264	288	254	271
272	288	270	279
237	249	248	248.5
211	220	221	220.5
180	192	192	192
201	214	208	211

$$e_{jt} = Y_t - X_{jt}, \qquad j = 1,2$$

such that

$$E[e_t] = 0, \qquad E[e^2_{jt}] = \sigma^2_j \qquad j = 1,2$$

and

$$E[e_{1t}e_{2t}] = \rho\sigma_1\sigma_2$$

where σ^2_j is the error variance of the j^{th} individual forecast and ρ is the correlation coefficient between the errors in the first set of forecasts and those in the second set.

Consider now a combined forecast, taken to be a weighted average of the two individual forecasts:

$$X_{ct} = kX_{1t} + (1 - k)X_{2t}$$

The forecast error is

$$e_{ct} = Y_t - X_{ct} = ke_{1t} + (1-k)e_{2t}$$

Hence the error variance is

$$\sigma_c^2 = k^2\sigma^2 + (1-k)^2\sigma_2^2 + 2k(1-k)\rho\sigma_1\sigma_2 \qquad \textbf{(1)}$$

This expression is minimized for the value of k given by:

$$k_0 = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

and substitution into equation (1) yields the minimum achievable error variance as:

$$\sigma_{c0}^2 = \frac{\sigma_1^2\sigma_2^2(1-\rho^2)}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

Note that $\sigma_{c0}^2 < \min(\sigma_1^2, \sigma_2^2)$ unless either ρ is exactly equal to σ_1/σ_2 or σ_2/σ_1. If either equality holds, then the variance of the combined forecast is equal to the smaller of the two error variances. Thus, a priori, it is reasonable to expect in most practical situations that the best available combined forecast will outperform the better individual forecast—it cannot, in any case, do worse.

Newbold and Granger (1974), Makridakis et al. (1982), Makridakis and Winkler (1983), Winkler and Makridakis (1983), and Makridakis and Hibbon (2000) have also reported empirical results that showed that combinations of forecasts outperformed individual methods.

Since Bates and Granger (1969), there have been numerous methods proposed in the literature for combining forecasts. However, the performance of different methods of combining forecasts varies from case to case. There is still neither definitive nor generally accepted conclusion that sophisticated methods work better than simple ones, including simple averages. As Clemen (1989) commented: *In many studies, the average of the individual forecasts has performed the best or almost best.* Others would agree with the comment of Bunn (1985) that the Newbold and Granger (1974) study and that of Winkler and Makridakis (1983) "demonstrated that an overall *policy* of combining forecasts was an efficient one and that if an *automatic* forecasting system were required, for example, for inventory planning, then a linear combina-

Table 2. Forecast errors and variances of errors

Errors of Individual Forecast 1	Errors of Individual Forecast 2	Errors of Combined Forecast
1	-3	-1
6	-10	-2
18	24	21
18	22	20
3	-9	-3
-17	-22	-19.5
-24	10	-7
-16	2	-7
-12	-11	-11.5
-9	-10	-9.5
-12	-12	-12
-13	-7	-10
Variance of errors = 196	**Variance of errors = 188**	**Variance of errors = 150**