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Qualitative Spatial Reasoning

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INTRODUCTION

Artificial Intelligence (AI) has, as one of its central topics, the ability to represent and reason with *common sense* knowledge. Early forays into common sense reasoning about the physical world involved solving textbook problems on physics and mathematics. These were not adequate for reasoning about most commonplace physical scenarios.

A system suggested by DeKleer, involving both quantitative knowledge and qualitative information concerning the physical situation marked the starting point for *qualitative physics* (Weld & DeKleer, 1990). Hayes' *Naive Physics Manifesto* (Hayes, 1985) paved the way for establishing qualitative physics (meantime re-christened *qualitative reasoning*) as an important topic of research within AI.

Qualitative Reasoning (QR) is an approach for dealing with common sense knowledge without recourse to complete quantitative knowledge. Representation of knowledge is through a limited repository of *qualitative abstractions*.

Space and spatial change is an important part of common sense reasoning. *Naive Physics Manifesto* proposed to represent space-time with four-dimensional *histories*. Despite early forays such as the *Naive Physics Manifesto*, representation of space within QR has been ill addressed. Nevertheless, there has been an increasing interest over the last few years in *qualitative spatial reasoning* - reasoning about space using qualitative abstractions.

BACKGROUND

Qualitative Spatial Representation and Reasoning is concerned with providing calculus which allow a machine to represent and reason with spatial entities without resort to traditional quantitative techniques. Reasoning is concerned with methods and techniques for decision-making using spatial knowledge and developing efficient algorithms for doing so. The term *Qualitative Spatial Reasoning* (QSR) subsumes both the sub-fields of representation and reasoning.

Conventional mathematical theories of space consider points as primitive spatial entities. Within QSR there is a strong tendency to take regions of space as the primitive spatial entity. The nature of the embedding space, that is, the universal spatial entity, is another important ontological commitment. One might take this to be R^n for some n, but one can imagine applications where discrete, finite or non-convex universes might be useful.

QUALITATIVE SPATIAL REASONING

Within QSR, *qualitative spatial representations* addressing different aspects of space including topology, orientation, shape, size, and distance have been put forward.

Different Approaches to QSR

Topology

Topology is the most elemental aspect of space and holds promise as a fundamental facet of qualitative spatial reasoning. Mathematical topology is too abstract to be of relevance to those attempting to formalize common sense spatial reasoning. QSR is concerned with reasoning and not just representation, and this has been paid little attention in mathematics.

One existing approach to topology, which has been espoused by QSR, is the work to be found in philosophical logic (Clarke, 1981; De Laguna, 1922). This work has built axiomatic theories of space which are predominantly topological in nature, and which take regions rather than points as primitive. In particular, the work of Clarke (1981) has led to the development of the *Region Connection Calculus* (Cohn, Bennett, Gooday, & Gotts, 1977; Randell, Cui & Cohn, 1992) and has also been a basis for theory of common sense geometry (Asher & Vieu, 1995).

Mereotopology

Mereology is the theory of parts and whole. Mereology is not sufficient by itself and there are theories in the literature, which have proposed integrating topology and mereology. The notion of *connection*, which is the key topological notion for the qualitative description of space, cannot be defined in terms of the mereological *part-whole* relation alone. Therefore, topological notions have to be added to mereology to provide an adequate qualitative theory of space. Such combination of the disciplines of mereology and topology is referred to as *mereotopology.*

Orientation

Orientation relations describe where objects are placed relative to one another. Of the qualitative orientation calculi to be found in the literature, certain calculi have an explicit triadic relation (Freksa, 1992), while others presuppose an extrinsic frame of reference (Frank, 1992).

Recently Dehak, Bloch, & Maitre (2005) have described a probabilistic method of inferring the position of a point with respect to a reference point knowing their relative spatial position to a third point. They address this problem in the case of incomplete information, where only the angular spatial relationships are known.

Distance and Size

Qualitative representation of distance is based on either some *absolute* scale or some kind of *relative* measurement. De Laguna's *Geometry of Solids* (De Laguna, 1922) is the earliest among the *relative* kind of representations. Distance is closely related to the notion of orientation, for example, distances cannot usually be summed unless they are in the same direction. It is perhaps not surprising that there have been a number of calculi, which are based on *positional information*: a primitive, which combines distance and orientation information.

Shape

Shape is an important characteristic of an object, and particularly difficult to describe qualitatively. Qualitative formalisms for describing shape can either be *constructive representations* or certain *constraining approaches*. Within the constructive representation of qualitative shape, structured combinations of primitive entities describe complex shapes. Approaches that work by describing the boundary of an object include sequence of different types of *curvature extrema* (Leyton, 1988). Meathrel & Galton (2000) present a general theory of qualitative outlines in 2D.

Region-Based Theories of Space

Early Theories

De Laguna's *Geometry of Solids* (De Laguna, 1922) is based on a triadic primitive CanConnect(x,y,z): x can connect y and z. CanConnect(x,y,z) is true if a body x can connect y and z by simple translation i.e., without scaling, rotation or shape change. The primitive is extremely expressive and it is easy to define notions such as *connectedness* and *relative distance* measures. Mereology as understood today is a formulation due to Tarski (1959) and is built on the single primitive relation P(x,y): x is a part of y. Tarski gave a theory of the *Geometry of Solids*, embedded by means of definition into an axiomatization of elementary Euclidean geometry.

Clarke's Calculus of Individuals

Clarke's formalism is based on connectedness (Clarke, 1981). Clarke took as his primitive C(x,y): the notion of two regions x and y being connected. C(x,y) is axiomatized to be reflexive and symmetric. From the C(x,y) relation, Clarke defines the relation of *part to whole* and several other useful spatial relations as enumerated in Table 1 below.

Region Connection Calculus

The Region Connection Calculus (RCC) is a modification and development of Clarke's original theory (Cohn, et al., 1997; Randell, et al., 1992). The basic part of the formal theory assumes a dyadic relation: C(x,y) to mean that region x is connected to region y.

The mereological relation of *parthood*, P(x,y) is defined from the connection relation C(x,y), which together is used to define a number of relations as enumerated in Table 2. DC(x,y) through NTPP(x,y) with the inverses for the last two, that is, TPP^{-1}(x,y) and $NTPP^{-1}$(x,y) constitute a *Jointly Exhaustive* and *Pair wise Disjoint* (JEPD) set of base relations referred to as RCC-8.

Table 1. Defined relations in Clarke's theory

Relation	Interpretation
DC(x,y)	x is disconnected from y
P(x.y)	x is part of y
PP(x,y)	x is proper-part of y
O(x,y)	x overlaps y
DR(x.y)	x is discrete from y
EC(x,y)	x is externally connected to y
TP(x.y)	x is a tangential part of y
NTP(x,y)	x is a non-tangential part of y