Efficient Multirate Filtering

Ljiljana D. Milić
University of Belgrade, Serbia

INTRODUCTION

A *multirate filter* can be defined as a digital filter in which the input data rate is changed in one or more intermediate points. With the efficient multirate approach, computations are evaluated at the lowest possible sampling rate, thus improving the computational efficiency, increasing the computation speed, and lowering the power consumption. Multirate filters are of essential importance for communications, image processing, digital audio, and multimedia. The role of multirate filtering in modern signal processing systems is threefold: Firstly, they are used whenever there is a need to preserve the signal properties when connecting two systems operating at different sampling rates. Secondly, multirate techniques are used for constructing filters with stringent spectral constraints that are very difficult, even impossible, to be solved otherwise. Thirdly, multirate filters are used in constructing multirate filter banks.

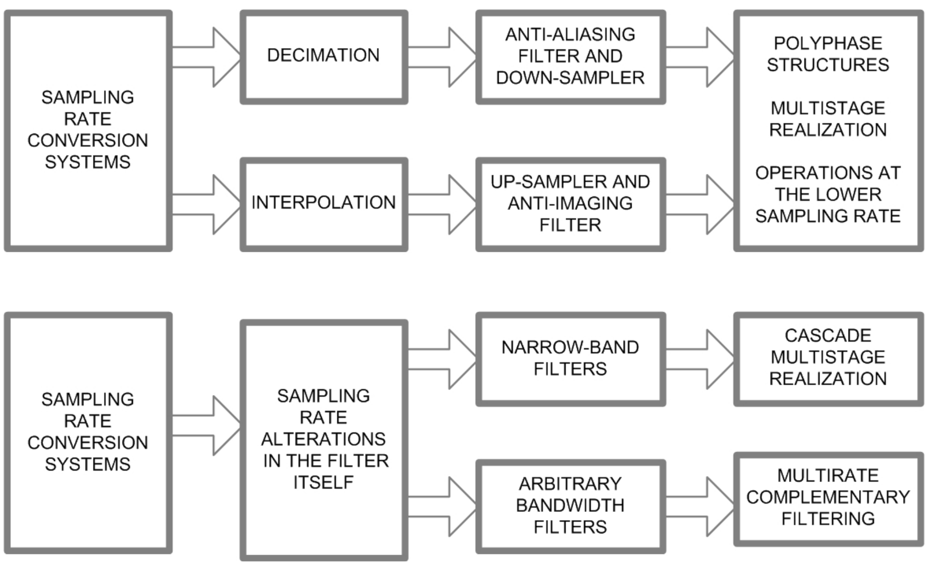
BACKGROUND

Efficient multirate filtering techniques have been developed during the past three decades for implementation of digital filters with stringent spectral constraints (Ansari & Liu, 1993; Bellanger, 1984, 1989; Crochiere & Rabiner, 1981, 1983; DeFata, Lucas & Hodgkiss, 1998; Fliege, 1994; Harris, 2004; Hentchel, 2002; Milić & Lutovac, 2002; Milić, Saramäki & Bregović, 2006; Mitra, 2006; Proakis & Manolakis, 1996; Vaidyanathan, 1990, 1993; Zelniker & Taylor, 1994).

MULTIRATE FILTERING TECHNIQUES

Multirate filtering is one of the best approaches for solving complex filtering problems when a single filter operating at a fixed sampling rate is of a very high order. With a multirate filter, the number of arithmetic operations per second

Figure 1. An overview of multirate filtering techniques



is considerably reduced. The multirate technique is used in filters for sampling rate conversion where the input and output rates are different, and also in constructing filters with equal input and output rates. For multirate filters, FIR (finite impulse response) or IIR (infinite impulse response) transfer functions can be used. An FIR filter easily achieves a strictly linear phase response, but requires a larger number of operations per output sample when compared with an equal magnitude response IIR filter. Multirate techniques significantly improve the efficiency of FIR filters that makes them very desirable in practice.

Figure 1 depicts an overview of different multirate filtering techniques.

Polyphase Realization

Polyphase realization is used to provide an efficient implementation of multirate filters. A *polyphase structure* is obtained when an Nth order filter transfer function is decomposed into M polyphase components, M<N. For FIR filters, polyphase decomposition is obtained simply by inspection of the transfer function (Crochiere, & Rabiner, 1983; Fliege, 1994; Harris, 2004; Mitra, 2006; Proakis & Manolakis 1996; Vaidyanathan, 1993). For multirate IIR filters, several approaches to polyphase decomposition have been developed (Bellanger, Bonnerot & Coudreuse, 1976; Crochiere, & Rabiner 1983; Drews & Gaszi, 1986; Krukowski & Kale, 2003; Renfors & Saramäki, 1987; Russel, 2000).

Multirate Filters for Sampling Rate Conversion

Filters are used in decimation to suppress aliasing, and in interpolation to remove imaging. The performance of the system for sampling rate conversion is mainly determined by filter characteristics. Since an ideal frequency response cannot be achieved, the choice of an appropriate specification is the first step in filter design.

Reducing the sampling rate by a factor of M is achieved by omitting every M-1 sample, or equivalently keeping every Mth sample. This operation is called down–sampling. In order to avoid aliasing, a low-pass *anti-aliasing filter* before down-sampling is needed. Therefore, a *decimator* is a cascade of an anti-aliasing filter and a down-sampler. To increase the sampling rate (interpolation by factor L), L-1 zeros are inserted between every two samples (up-sampling). An interpolation filter has to be used to prevent imaging in the frequency band above the low-pass cutoff frequency. An *interpolator* is a cascade of an up-sampler and an *anti-imaging* filter.

The efficiency of FIR filters for sampling rate conversion is significantly improved using the *polyphase realization*. Filtering is embedded in the decimation/interpolation process and a *polyphase structure* is used to simultaneously achieve the interpolation/decimation by a given factor but running at a low data rate.

Due to the *polyphase multirate implementation*, the number of arithmetic operations in linear-phase FIR filters is decreased by a factor M (or L). An effective method, which leads to high efficiency for a high-order FIR filter is proposed in Muramatsu & Kiya (1997). Efficient decimation and interpolation for the factor M=2 (L=2) is achieved with FIR half-band filters since the number of constants is a half of the filter length.

Very sharp filters with reduced computational efficiency can be achieved by combining the multirate approach and frequency response masking techniques (Lim & Yang, 2005).

Polyphase IIR filters require lower computation rates among the known decimators and interpolators (Renfors & Saramäki, 1987). If a strictly linear phase characteristic is not requested, an IIR filter is an adequate choice. Moreover, an IIR transfer function can be designed to approximate a linear phase in the pass-band (Jaworski & Saramäki, 1994; Lawson, 1994; Surmo-Aho & Saramäki, 1999). An IIR decimator or interpolator is particularly useful in applications that cannot tolerate a considerably large delay of an adequate FIR decimator or interpolator. For a restricted class of filter specifications, an attractive solution based on all-pass subfilters can be used leading to very efficient implementation (Krukowski & Kale, 2003; Renfors & Saramäki, 1987). The most attractive solution is an IIR half-band filter implemented with two all-pass subfilters (Johansson & Wanhammar, 1999; Krukowski & Kale, 2003; Milić & Lutovac, 2002; Renfors & Saramäki, 1987). For a rational conversion factor L/M a very efficient decomposition of IIR filter is proposed in Russel (2000).

There are many applications requiring the sampling rate conversion between arbitrary sampling rates. The sampling rate alteration by an arbitrary factor can be viewed as the computation of the new sample values at arbitrary time instants between the existing samples. There are many valuable contributions in the literature, which concentrate on the various solutions of this important problem (Harris, 2004; Mitra, 2006; Vesma & Saramäki, 2007).

Multirate Filters with Equal Input and Output Rates

Digital filters with sharp transition bands are difficult, sometimes impossible, to be implemented using conventional structures. A serious problem with a sharp FIR filter is its complexity. The FIR filter length is inversely proportional to transition–width and complexity becomes prohibitively high for sharp filters (Crochiere & Rabiner, 1983; Fliege, 1994; Mitra, 2006; Proakis & Manolakis, 1996; Saramäki, 1993; Vaidyanathan, 1993). In a very long FIR filter, the