Chapter 7
Data Visualization Using Weighted Voronoi Diagrams

Raghuveer Devulapalli
University of Minnesota, USA

Neil Peterson
University of Minnesota, USA

John Gunnar Carlsson
University of Southern California, USA

ABSTRACT

A Voronoi diagram is a standard spatial tessellation that partitions a domain into sub-regions based on proximity to a fixed set of landmark points. In order to maintain control over the size and shape of these sub-regions, a weighting scheme is often used, in which each landmark has a scalar value associated with it. This suggests a natural "inverse" problem: given a fixed set of landmark points in a given planar region and a set of "desired" areas, is it possible to calculate a set of weights so that each sub-region has a particular area? In this chapter, the authors give a fast scheme for determining these weights based on theory from convex optimization, which is then applied to a variety of problems in data visualization.

INTRODUCTION

A *Voronoi diagram* of a planar region $\mathcal{R}$ and a set of points $\mathcal{P} = \left\{ p_1, \ldots, p_n \right\} \subset \mathcal{R}$ is a tessellation of $\mathcal{R}$ into n sub-regions $\left\{ R_1, \ldots, R_n \right\}$ based on proximity to the

nearest point in $\mathcal{P}$:

$$R_i := \left\{ x \in \mathcal{R} : \| x - p_i \| \leq \| x - p_j \| \forall j \right\}.$$

The norm $\| \cdot \|$ in the above expression is usually Euclidean, but may also be taken under the L^1 or L^∞ norm, for example (Lee & Wong, 1980). Voronoi diagrams are widely applied in many different domains, such as data compression, facility location, robotics, and air traffic control, to name a few. Indeed, such structures are useful and intuitive for modelling many different kinds of spatial interactions: for example, if the points represent stores that are otherwise identical, then the Voronoi diagram describes the market regions for those stores, inasmuch as customers will prefer whichever store is closest to them. Voronoi diagrams are also of fundamental importance in classification: if the points represent reference data such as documents or images, then the Voronoi diagram describes the set of all "query" data that are most closely associated with a particular reference point. Figure 1a shows an example of a Voronoi diagram.

Voronoi diagrams have been generalized in a number of ways by introducing a *weight vector* $\mathbf{w} = \left\{ w_1, \ldots, w_n \right\}$ associated with the landmark points (Aurenhammer et al., 2013; Okabe et al., 2009); the tessellation $\left\{ R_1, \ldots, R_n \right\}$ then depends on these weights as well. The authors per this chapter consider three weighting schemes:

- In an *additively weighted Voronoi diagram*, the tessellation satisfies

$$R_i := \left\{ x \in \mathcal{R} : \| x - p_i \| - w_i \leq \| x - p_j \| - w_j \forall j \right\};$$

it can be shown that the boundaries between sub-regions are always hyperbolic arcs and that all sub-regions are connected.

- In a *multiplicatively weighted Voronoi diagram*, the tessellation satisfies

$$R_i := \left\{ x \in \mathcal{R} : \frac{\| x - p_i \|}{w_i} \leq \frac{\| x - p_j \|}{w_j} \forall j \right\};$$

it can be shown that the boundaries between sub-regions are always circular arcs.

22 more pages are available in the full version of this document, which may be purchased using the "Add to Cart" button on the publisher's webpage: www.igi-global.com/chapter/data-visualization-using-weighted-voronoi-diagrams/136104

Related Content

Multcriteria Decision Analysis for Collaborative GIS
Jacek Malczewski (2006). *Collaborative Geographic Information Systems (pp. 167-185).*
www.irma-international.org/chapter/multcriteria-decision-analysis-collaborative-gis/6658

Virtual Environments for Geospatial Applications
Magesh Chandramouliand Bo Huang (2013). *Geographic Information Systems: Concepts, Methodologies, Tools, and Applications (pp. 216-226).*
www.irma-international.org/chapter/virtual-environments-geospatial-applications/70443

Integrated Ontologies for Spatial Scene Descriptions
Sotirios Batsakisand Euripides G.M. Petrakis (2012). *Qualitative Spatio-Temporal Representation and Reasoning: Trends and Future Directions (pp. 321-335).*
www.irma-international.org/chapter/integrated-ontologies-spatial-scene-descriptions/66764

Leveraging Volunteered Geographic Information to Improve Disaster Resilience: Lessons Learned From AGORA and Future Research Directions
João Porto de Albuquerque, Flávio Eduardo Aoki Horita, Livia Castro Degrossi, Roberto dos Santos Rocha, Sidgley Camargo de Andrade, Camilo Restrepo-Estradaand Werner Leyh (2017). *Volunteered Geographic Information and the Future of Geospatial Data (pp. 158-184).*
www.irma-international.org/chapter/leveraging-volunteered-geographic-information-to-improve-disaster-resilience/178804

The Perceived Role of Communities and Government Officials in Solid Waste Management in Ghana, West Africa
Pearl Sika Fichteland Leslie A. Duram (2022). *International Journal of Applied Geospatial Research (pp. 1-18).*
www.irma-international.org/article/the-perceived-role-of-communities-and-government-officials-in-solid-waste-management-in-ghana-west-africa/295863