Comb Filters Characteristics and Applications

Miriam Guadalupe Cruz Jimenez
National Institute of Astrophysics, Optics and Electronics (INAOE), Mexico

David Ernesto Troncoso Romero
National Institute of Astrophysics, Optics and Electronics (INAOE), Mexico

Gordana Jovanovic Dolecek
National Institute of Astrophysics, Optics and Electronics (INAOE), Mexico

INTRODUCTION

A comb filter is a Linear Time-Invariant (LTI) *digital filter*, where linear means that its output to a *scaled* sum of input *digital signals* is equal to the scaled sum of the outputs to every one of these input signals (i.e., the filter satisfies the superposition principle) and time-invariant means that, for any input signal that has a given *delay*, the output undergoes the same delay as the input (Antoniou, 2006). The name comb is derived by the fact that its *magnitude response* resembles the teeth of a comb. Since there are several filters having magnitude responses with such characteristic, the term comb filter is rather general. The duration of the *impulse response* of comb filters can be either finite or infinite, i.e., there are Finite Impulse Response (FIR) comb filters and Infinite Impulse Response (IIR) comb filters (Zölzer, 2008).

The simplest FIR comb filter has the following *transfer function*,

$$H_a(z) = 1 + z^{-M}. \qquad (1)$$

This filter adds to a signal a version of that signal delayed by *M* sample periods, and it is the basic building block to introduce echo effects in audio signals (Zölzer, 2008). Moreover, if the addition in (1) is replaced by a subtraction, the resulting comb filter is a useful building block to remove DC and harmonics (Diniz, Da Silva & Neto, 2010). The unintentional delay of an audio signal due to the environment is also modeled as a comb filter (Toole, Shaw, Daigle & Stinson, 2001), and this effect may be undesirable

in many cases. Similarly, a simple IIR comb filter has the following transfer function,

$$H_b(z) = 1/(1 - az^{-M}), \qquad (2)$$

with $a < 1$. This filter is a basic building block to model and create reverberation effects or, in general, to artificially reproduce the acoustics of a room (Zölzer, 2008).

One of the most important comb filters for several Digital Signal Processing (DSP) applications is the one based in the FIR filter where all the samples of its impulse response have values equal to one. Unlike the aforementioned comb filters described by $H_a(z)$ in (1) and $H_b(z)$ in (2), this comb filter has a low-pass characteristic, which makes it useful to pass a baseband signal and remove unwanted high-frequency spectra (Milic, 2009). The rest of this article is dedicated to this particular filter, which will be referred as the comb filter hereafter. The main characteristics of the comb filter, as well as its advantages and disadvantages will be highlighted. Moreover, we will present the selected methods commonly used to decrease the disadvantages of the comb filters with minimum affectation of its advantages.

In the efficient implementation of the comb filter, a comb filter with transfer function based in (1) (just with the addition replaced by a subtraction) is employed. In order to avoid confusion, that filter will be referred hereafter as comb differentiator, since it is based on a simple first-order differentiator (Regalia, 1993).

Figure 1. Filtering structures: (a) FIR comb filter in non-recursive structure and (b) recursive comb filter with an integration part and a comb differentiation part, where the delay z^{-1} has been replaced by z^{-M}
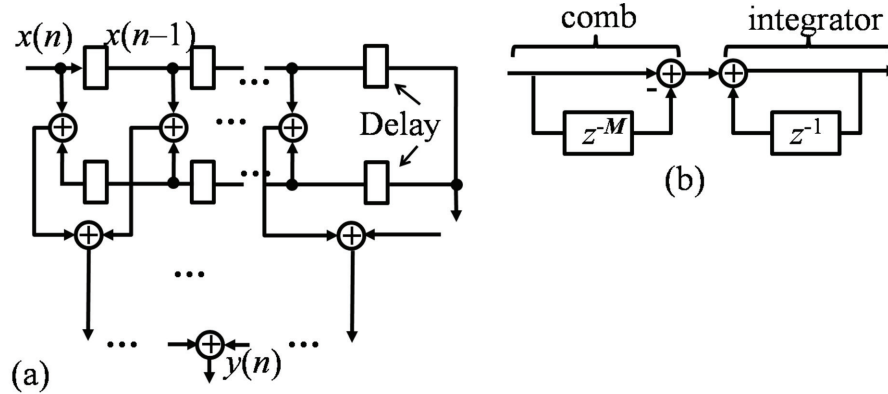
BACKGROUND

Consider a simple FIR filter that has the following transfer function (Milic, 2009),

$$H(z) = \frac{1}{M} \sum_{k=0}^{M} z^{-k}, \qquad (3)$$

where M is the filter order. The coefficients that multiply the variable z are all equal to 1. Thus, the non-recursive implementation of this filter does not require *multipliers* for its coefficient's values. The impulse response of this filter is $h(k) = 1$ for $0 \leq k \leq M$ and 0 for other values of k. The scaling factor $(1/M)$ is included to provide a normalized gain of 0 dB at frequency equal to zero.

The transfer function of the comb filter arises from expressing the transfer function given in (3) in recursive form as follows (Lyons, 2004),

$$H(z) = \frac{1}{M} \frac{1 - z^{-M}}{1 - z^{-1}}. \qquad (4)$$

This transfer function was first used in (Hogenauer, 1981) as starting point for an efficient implementation of the filtering required in systems where the sampling rate is increased (upsampling) or decreased (downsampling). Both, the non-recursive structure to implement (3) and the recursive structure to implement (4) are shown in Figure 1. In the context of digital filters, non-recursive means that the filtering structure does not require a feedback, whereas recursive means that

the filtering structure has a feedback (i.e., the output depends on delayed versions of that output). Since the transfer function (3) and (4) are equivalent, the comb filter is, in fact, the same. We observe from (4) that the recursive transfer function is the product of the terms $1/(1-z^{-1})$, which is an integrator, and $(1-z^{-M})$, which is a differentiator whose delay has been replaced by M delays. The frequency response of the comb filter is

$$H(e^{j\omega}) = \frac{1}{M} \frac{\sin(\omega M / 2)}{\sin(\omega / 2)} e^{-j\omega(M-1)/2}, \qquad (5)$$

where ω is the normalized angular frequency.

The comb filter is very popular because it has the following advantages:

- It is a FIR recursive system, so it has linear-phase and guaranteed stability whenever the proper bus width is used (Lyons, 2004).
- It is multiplier-free and it only requires two additions (in fact they are subtractions, but *adders* and subtracters are considered with equal hardware complexity). Since *multipliers* are the most expensive elements in a filter, the comb filter can be used to design low-complexity multiplierless filters (Lyons, 2004).
- In the frequency range from $\omega=0$ to $\omega=\pi$, the zeros of the magnitude response of a comb filter are placed over the frequencies $\omega_k = 2\pi k/M$, for $k = 1, 2, \ldots, \lfloor M / 2 \rfloor$ ($\lfloor x \rfloor$ denotes the inte-

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