Efficient Techniques to Design Low–Complexity Digital Finite Impulse Response (FIR) Filters

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INTRODUCTION

A discrete-time signal is a sequence of numbers, so-called samples. A sample occurs at T_s seconds, then another sample appears during the next T_s seconds and so on, forming the sequence. The time T_s is called sampling period and the number produced in the n-th sampling period (i.e., after n times T_s seconds) is denoted as $x(nT_s)$. For analysis purposes it is usual to assume that $x(nT_s) = 0$ for negative values of n. In general, the samples can be *complex numbers*. If so, the signal is a complex signal. In these analyses, the digital frequency f is a value that expresses how much of a cycle of a sinusoidal wave is represented by a sample. In general terms, discrete-time signals may contain information for different values of f.

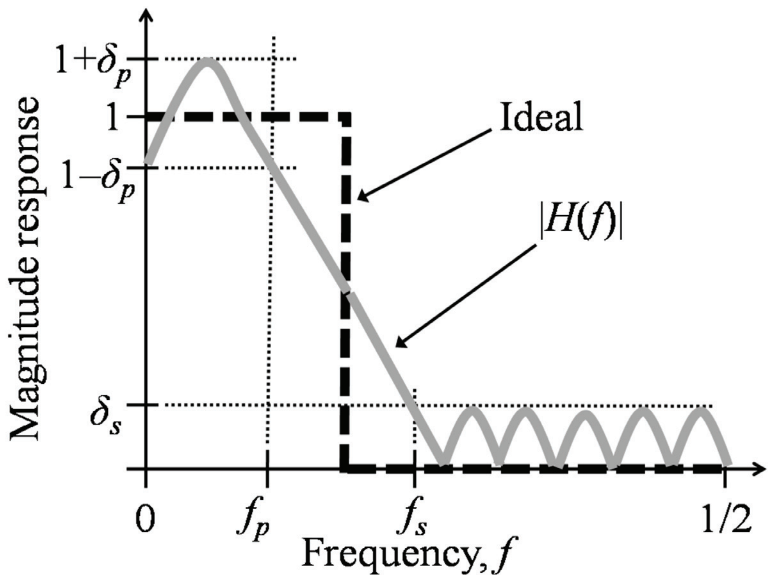
A digital filter is a system that receives a discrete-time signal and returns another discrete-time signal with modified characteristics (Antoniou, 2006). A useful characteristic that many digital filters exhibit is the *Linear Time-Invariant* (LTI) property. When the input signal $x(nT_s)$ has the value 1 for $n = 0$ and 0 for other values of n, the output signal of a LTI digital filter is called impulse response. LTI digital filters are often classified by the duration of its impulse response as Infinite-duration Impulse Response (IIR) or Finite-duration Impulse Response (FIR) filters (Proakis & Manolakis, 1996). Additionally, a LTI digital filter is generally described by its frequency response, which is the response of the filter to a *complex exponential signal* with frequency $2\pi f$. The frequency response is a *complex function* of f and is periodic over f, with a period of $1/T_s$. Because of this periodicity, the frequency response is just observed in the range of values from $f = -1/2T_s$ to $f = 1/2T_s$. As a complex function, the frequency response has two functions associated with it: its *magnitude response* and its *phase response*.

When the values of the impulse response are real, the magnitude response is symmetric and the phase response is anti-symmetric around $f = 0$ (Saramaki, 1993). Thus, these responses are just observed in the range of values from $f = 0$ to $f = 1/2T_s$. Systems with real impulse response are used in a wide variety of applications, being one of the most common the traditional low-pass filter. This filter passes the frequency components of the input signal that range from 0 to f_p and rejects the frequency components of the signal that range from f_s to $1/2T_s$, with $0 < f_p < f_s < 1/2T_s$. The values f_p and f_s are the passband edge and the stopband edge frequencies, respectively, whereas the range from 0 to f_p is the passband, the range from f_s to $1/2T_s$ is the stopband and the difference $f_s - f_p$ is the transition band. The ideal values of the magnitude response in passband and stopband are, respectively, 1 and 0. For a realizable filter, an acceptable deviation from these ideal values must be specified. Usually, these deviations are represented as the numbers δ_p and δ_s, respectively. Figure 1 shows the specifications of the magnitude response of a low-pass filter, along with the ideal response and the actual magnitude response of the filter, denoted as $|H(f)|$. For analysis purposes it is usual to assume $T_s = 1$, i.e., $x(nT_s) = x(n)$. Thus, in Figure 1 the range of interest of values f is from 0 to 1/2.

Digital FIR filters play a central role in modern Digital Signal Processing (DSP) systems. These filters find extensive applications in communication systems, audio systems, instrumentation, image enhancement, processing of geophysical signals, processing of bio-

DOI: 10.4018/978-1-4666-5888-2.ch151

Figure 1. Magnitude response specifications of a low-pass filter



logical signals, among others (Antoniou, 2006).This is mainly due to the advantages of FIR filters, being the most important (Saramaki, 1993):

- Absolute stability.
- The fact that FIR filters can be designed to have linear phase response.

The main disadvantage of conventional FIR filter designs is that they require, especially in applications demanding narrow transition bands, a high number of arithmetic operations and hardware components. This makes the implementation of FIR filters with narrow transition bands very costly (Saramaki, 1993). However, a number of methods to design low-complexity FIR filters have been developed over the past three decades, approximately. One of the most popular approaches among them consists on designing an overall filter as an interconnection of simple subfilters, where some of these subfilters have a frequency response with one or more of its periodic shapes, so-called images, appearing into the range $0 < f < 1/2$ (in other words, the period of the frequency response of these subfilters is 1/2 or smaller). Methods based on this approach are adequate from a low power consumption perspective, since the subfilters are simple and the overall filter performs a lower number of computations in comparison to a direct design. These methods are classified as "methods based on periodic subfilters" (Saramaki, 1993).

The design of low-complexity FIR filters is a challenging task since many aspects have to be taken into account. The efficient design of FIR digital filter needs a trade-off between very stringent design specifications, low power consumption, low area requirements, high speed of computations and low time and design effort. An acceptable design should balance the trade-off to a reasonable degree. Thus, in this article we introduce a review of methods based on periodic subfilters to efficiently design low-complexity linear-phase FIR filters.

BACKGROUND

The output signal $y(n)$ as of a LTI FIR filter is given as,

$$y(n) = \sum_{k=0}^{N} h(k)x(n-k), \qquad (1)$$

where the integer N is the filter order and the values $h(k)$, with $0 \leq k \leq N$, are the samples of the impulse response of the filter. A FIR filter has a linear phase if its impulse response satisfies either $h(k) = h(N-k)$, which is called condition of symmetry, or $h(k) = -h(N-k)$, which is called condition of anti-symmetry, for $0 \leq k \leq N$. Figure 2 illustrates the impulse response of a symmetric filter for N even.