Linear Programming

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INTRODUCTION

Linear programming (LP or linear optimization) deals with the problem of the optimization (minimization or maximization), in which a linear objective function is optimized subject to a set of linear constraints. Its name means that planning (programming) is being done with a mathematical model. It is one of widely used techniques in operations research and management science. Some typical applications are: 1. a manufacture wants to develop a production schedule and an inventory policy that will satisfy sales demand in future periods. Ideally, the schedule and policy will enable the company to satisfy demand and at the same time *minimize* the total production and inventory costs. 2. A financial analyst must select an investment portfolio from a variety of stock and bond investment alternatives. The analyst would like to establish the portfolio that *maximizes* the return on investment. 3. A marketing manager wants to determine how best to allocate a fixed advertising budget among alternative advertising media such as radio, television, newspaper, and magazine. The manager would like to determine the media mix that *maximizes* advertising effectiveness. 4. A company has warehouse in a number of locations throughout a country. For a set of customer demands, the company would like to determine how much each warehouse should ship to each customer so that total transport costs are *minimized*. These examples are only a few of situations in which linear programming has been used successfully, but illustrate the diversity of linear programming applications (Anderson et al., 2011). Other applications include supply chain management in the motor industry, productions scheduling in the brewing industry, aircraft crew and production scheduling, financial planning and capital budgeting, asset and liability management, energy management in the utilities sector, network design in the telecommunications sector and transportation (Dowman & Wilson, 2002).

Linear programming, a specific case of mathematical programming, has some properties in common. Linear programs can be expressed in canonical form:

Maximize $c^T x$

Subject $Ax \leq b$

and $x \geq 0$

We are given an m-vector, $b = (b_1, \ldots, b_m)$, an n-vector, $c = (c_1, \ldots, c_m)$, and an m × n matrix A, where x represents the vector of decision variables (to be determined), c and b are vectors of coefficients, A is a matrix of coefficients, and $(\cdot)^T$ is the matrix transpose. The expression to be maximized or minimized is called the objective function ($c^T x$ in this case). The inequality $A\,x$ ≤ b is the constraint which specifies a convex polytope over which the objective function is to be optimized. The column vector b which is referred to as the right-hand-side vector represents the maximal requirements to be satisfied. A set of variables x satisfying all the constraints is called feasible point or a feasible vector. The set of all such points constitutes the feasible region. Using the foregoing terminology, the linear programming problem can be stated as follows: among all feasible regions, find one that minimizes (or maximize) the objective function. It is fairly common for large linear programming models to in-

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clude a mixture of functional constraints, some with " $\leq$ "signs, some with " $\geq$ "signs, and some with " $=$" signs.

Many people have contributed to the growth of linear programming by developing its mathematical theory, devising efficient computational methods and codes, exploring new algorithms and new applications, and by their use of linear programming as an aiding tool for solving more complex problems, for instance, discrete programs, nonlinear programs, combinatorial problems, stochastic programming problems and problems of optimal control.

This article first presents the background of linear programming and then focuses on concepts, characteristics, techniques, general theories, and effective solution algorithms of linear programming optimization problems. The simplex algorithm provides considerable insights into the theory of linear programming and yields an efficient algorithm in practice. It also presents Karmarkar's polynomial-time algorithm for linear programming problems because it compares favorably with the simplex method, particularly for general large-scale problems. At last, it indicates future trends and makes a conclusion.

BACKGROUND

Linear programming problem was first conceived by George B. Danzig around 1947 while he was first mathematical advisor to the United States Air Force Comptroller on developing a mechanized planning tool for a time-staged development, training, and logistical supply program. Soviet mathematician and economist Leonid Kantorovich developed the earliest linear programming problems in 1939 for use during World War II to plan expenditures and returns in order to reduce costs to the army and increase losses to the enemy. But his work remains unknown until 1959. Hence the conception of the general classes of linear programming problem is usually credited to Danzig. Because the Air Force refers to its various plans and schedules to be implemented as "programs," the method was kept secret until 1947 when George B. Dantzig published the simplex method and John von Neumann developed the theory of duality as a linear optimization solution, and applied it in the field of game theory. Danzig's first published paper addressed this problem as "Programming in a Linear Structure." The term linear programming was actually coined by the economist and mathematician T.C. Koopmans in the summer of 1948 while he and Danzig strolled near the Santa Monica beach in California (Bazaraa et al., 2009).

Due to the wide applicability of linear programming models, an immense amount of work has appeared regarding theory and algorithms for LP. During and after World War II it became evident that planning and coordinating among various projects and the efficient utilization of scarce resources were essential. Interest in linear programming spread quickly among industrial engineering, management, operations research, computer science, economics, mathematicians, statistics, and government institutions.

Linear programming is a considerable field of optimization for several reasons. Many practical problems in operations research and management science can be expressed as linear programming problems. Certain special cases of linear programming, such as network flow problems and multi-commodity flow problems are considered important enough to have generated much research on specialized algorithms for their solution. Historically, ideas from linear programming have inspired many of the central concepts of optimization theory, such as duality, decomposition, and the importance of convexity and its generalizations. Although the modern management issues are ever changing, most companies would like to maximize profits or minimize costs with limited resources. Therefore, many issues can be characterized as linear programming problems. It is not surprising that in a recent survey of Fortune 500 companies, 85% of those responding said that they had used linear programming (Winston & Albright, 2012). The history, theory, and applications of linear

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