FGP for Chance Constrained Fractional MODM Problem

F

Shyamal Sen
Brahmananda Keshab Chandra College, India

Bijay Baran Pal
University of Kalyani, India

INTRODUCTION

In a real-world decision situation, decision makers (DMs) are often faced with the problem of setting parameter values owing to imprecision in human judgments as well as inherent uncertainty in parameter values of problems. The two types of prominent approaches for solving such problems are: stochastic programming (SP) (Dantzig, 1955) which deals with probabilistic uncertain data and fuzzy programming (FP) (Zimmermann, 1978) which deals with fuzzily described data.

SP is a branch of mathematical programming, where some / all of model parameters of a problem are random in nature. The Chance constrained programming (CCP) as a special field of SP (Charnes, & Cooper, 1959) has been studied deeply and used effectively to a real-life problem (Liu, Wu, & Hao, 2012; Mesfin, & Shuhaimi, 2010). On the other hand, FP based on the theory of fuzzy sets (Zadeh, 1965) has been studied (Zimmermann, 1987) extensively in the past and employed to different real-life problems (Li, Xu, & Gen, 2006; Slowinski, 1986). Further, fuzzy goal programming (FGP) (Pal, & Moitra, 2003) based on the notion of goal satisficing philosophy (Simon, 1957) in goal programming (GP) (Ignizio, 1976) has also been studied in the past, and applied to various problems (Kumar, & Pal, 2013; Pal, & Chakraborti, 2013) in the recent past.

In this chapter, a parametric programming based solution approach, initially introduced by Dinkelbach (1967), is addressed to solve chance constrained multiobjective decision making (MODM) problems with fractional criteria. In the proposed approach, the linear forms of the defined deterministic equivalents of chance constraints with continuous random parameters are considered to solve the problem by employing *parametric minsum* FGP methodology. In the solution process, minimization of under-deviational variables associated with membership goals of the defined fuzzy goals according to their relative weights of importance is considered to arrive at optimal decision in imprecise environment.

BACKGROUND

The fractional programming (Schaible, & Ibaraki, 1983) with multiplicity of objectives have been studied (Steuer, 1986) previously as a special field of study in the area of nonlinear programming (NLP) (Avriel, 1976), where objectives appear in the form of ratios in the programming and planning environment. The deep study in the area of fractional programming has been made in the past and extensively appeared (Craven, 1988; Zhu, & Huang, 2011) in the literature.

The methodological aspects of solving fuzzily described multiobjective fractional programming problems (MOFPPs) have been studied (Rommelfanger, Hanuscheck, & Wolf, 1989; Pal, Moitra, & Maulik, 2003) in the past and well documented in the literature. The linearization method with the use of variable changes for solving MOFPPs has

also been suggested (Pal, Moitra, & Sen, 2011) in the past. However, the extensive study in this area is still at an early stage.

Now, the mathematical framework of a chance constrained linear MOFPP is presented in the following section.

MODEL FORMULATION

The general format of a chance constrained linear MOFPP can be stated as:

Find $X(x_1, x_2, ..., x_n)$ so as to:

Maximize $Z_k(X) = \dfrac{G_k X + \alpha_k}{H_k X + \beta_k}, \quad k \in K_1,$

Minimize $Z_k(X) = \dfrac{G_k X + \alpha_k}{H_k X + \beta_k}, \quad k \in K_2,$

subject to

$$X \in S\{X \in R^n \mid Pr[AX \begin{pmatrix} \geq \\ \leq \end{pmatrix} b] \geq p, \tag{1}$$

$$X \in R^n, \ X \geq 0, \ b \in R^m\}$$

where *Pr* indicates the probabilistically defined constraints, $A = (a_{ij})_{m \times n}$ is a coefficient matrix and b is a resource vector, $G_k = (g_{k1}, g_{k2}, ..., g_{kn})$ and $H_k = (h_{k1}, h_{k2}, ..., h_{kn})$ are the coefficient vectors and where α_k and β_k are constants and $p(0<p<1)$ is the vector of satisficing probability levels defined for randomness of parameters associated with the constraints set. It is assumed that the feasible region S is nonempty ($S \neq \varphi$), and where

$$K_1 \cup K_2 = \{1, 2, ..., K\}, K_1 \cap K_2 = \varphi.$$

Now, it is assumed that the parameters are independent continuous normally distributed random variables. Then, conversion of the chance constraints in (1) into deterministic equivalents is described in the following section.

Deterministic Equivalents of Chance Constraints

The chance constraints set in (1) with $\geq$ type can be explicitly presented as (Blumenfeld, 2010):

$$Pr[\sum_{j=1}^{n} a_{ij}x_j \geq b_i] \geq p_i, \quad i = 1, 2, ..., m_1, \quad m_1 < m \tag{2}$$

Let $E(a_{ij})$ and $E(b_i)$, and $Var(a_{ij})$ and $Var(b_i)$ be the *means* and *variances* of the associated random variables a_{ij} and b_i with the characteristics of *normal distribution*, where $E(.)$ and $Var(.)$ stand for mean and variance, respectively.

Then, in the sequel of deterministic conversion, let $F_i(y)$ be the distribution function of the *i*-th random variable b_i. Since $F_i(y)$ is a monotonically non-decreasing function, the value of corresponding variable is determined as

$$F_i^{-1}(\varepsilon) = \{Max\ y\ /\ Pr(b_i \leq y) \leq \varepsilon\}, \quad 0 < \varepsilon < 1 \tag{3}$$

Here, since a_{ij} and b_i are normally distributed random variables, the conversion process can be described as follows.

Let,

$$y_i = \sum_{j=1}^{n} a_{ij}x_j - b_i \tag{4}$$

Since, y_i is linear combination of normally distributed random variable; it would also follow the characteristics of normal distribution.

Related Content

Modeling Energy Portfolio Scoring: A Simulation Framework
Rafael Diaz, Joshua G. Behr, Rafael Landaeta, Francesco Longoand Letizia Nicoletti (2015). *International Journal of Business Analytics (pp. 1-22).*
www.irma-international.org/article/modeling-energy-portfolio-scoring/132799

Do Users Go Both Ways?: BI User Profiles Fit BI Tools
Hamid Nemati, Brad Earle, Satya Arekapudiand Sanjay Mamani (2012). *Organizational Applications of Business Intelligence Management: Emerging Trends (pp. 209-227).*
www.irma-international.org/chapter/users-both-ways/63976

Deep Learning of Data Analytics in Healthcare
Yang Lu (2020). *Theory and Practice of Business Intelligence in Healthcare (pp. 151-165).*
www.irma-international.org/chapter/deep-learning-of-data-analytics-in-healthcare/243354

Knowledge Generation Using Sentiment Classification Involving Machine Learning on E-Commerce
Swarup Kr Ghosh, Sowvik Deyand Anupam Ghosh (2019). *International Journal of Business Analytics (pp. 74-90).*
www.irma-international.org/article/knowledge-generation-using-sentiment-classification-involving-machine-learning-on-e-commerce/226973

First Look on Web Mining Techniques to Improve Business Intelligence of E-Commerce Applications
G. Sreedharand A. Anandaraja Chari (2017). *Handbook of Research on Advanced Data Mining Techniques and Applications for Business Intelligence (pp. 298-314).*
www.irma-international.org/chapter/first-look-on-web-mining-techniques-to-improve-business-intelligence-of-e-commerce-applications/178114