Copula-Based Multivariate Time Series Models

Iva Mihaylova
University of St. Gallen, Switzerland

INTRODUCTION

It has been more than fifty years since Abe Sklar introduced the concept "copula", meaning "a connection", in the context of probabilistic metric spaces. In the last decade, copulas have become a widely used statistical tool for dependence modeling applications. Genest, Gendron, & Bourdeau-Brien, (2009) reported 750,000 Google hits of the word "copula" in 2009, compared to 10,000 in 2003 (Mikosch, 2006).

The popularity of copulas is explained by the growing evidence (see, e.g., Jondeau, Poon, & Rockinger, 2007) that the dependence among the financial and economic phenomena is incompatible with the traditionally applied (multivariate) Gaussian distribution and coefficient of linear correlation. Finding an appropriate multivariate perspective of a concrete dependence modeling task without resorting to copulas may be challenging or even impossible, because the set of available parametric multivariate distributions is considerably smaller than the set of parametric univariate distributions. As commented by Joe (1997), the "study of multivariate distributions is not easy because one cannot just write down a family of functions and expect it to satisfy the necessary conditions for multivariate cumulative distribution functions [...]." (p. 3). Sklar's (1959) theorem has defined the copula concept and laid the theoretical foundation for a separate, two-stage modeling of the stochastic variables' marginal distributions and their joint dependence structure.

The usefulness of copulas consists in the fact that any of them can link any valid marginal distribution into a valid multivariate distribution. Consequently, it can be stated that copulas expand the set of possible (not automatically meaning "appropriate") multivariate distributions and increase the probability of finding an optimal dependence model for the concretely analyzed data. Due to their flexibility, copulas have become a preferred statistical tool when the research goal is to explore dependence. As an influential earlier contribution can be mentioned Embrechts, McNeil, & Straumann (1999). A detailed overview of copula methods is provided in the books of Joe (1997) and Nelsen (2006), while the emphasis in McNeil, Frey, & Embrechts (2005) is on risk management. In the area of finance, Cherubini, Luciano, & Vecchiato (2004) concentrate predominantly on copula applications for derivatives pricing and credit risk analysis, complemented by Cherubini, Mulinacci, Gobbi, & Romagnoli (2011) with new convolution-based copulas.

Sklar (1959) defines the copula concept only for the static (unconditional) case. However, there is well-documented evidence that the dependence measures of observed phenomena in economics and finance have a time-varying behavior (see, e.g., Engle, 2002). Patton (2006) has proposed a conditional copula, an extension of Sklar's theorem, which incorporates both marginal and joint conditional distributions, as well as a parametric model for the dynamics of the copula dependence parameter. This new copula model can describe the time-varying dependence of multivariate time series data, the main theme of the present chapter. Its central goal is to briefly introduce the most popular static copulas in finance and economics, as well as to concentrate on recent advances and trends, concerning time-varying copulas.

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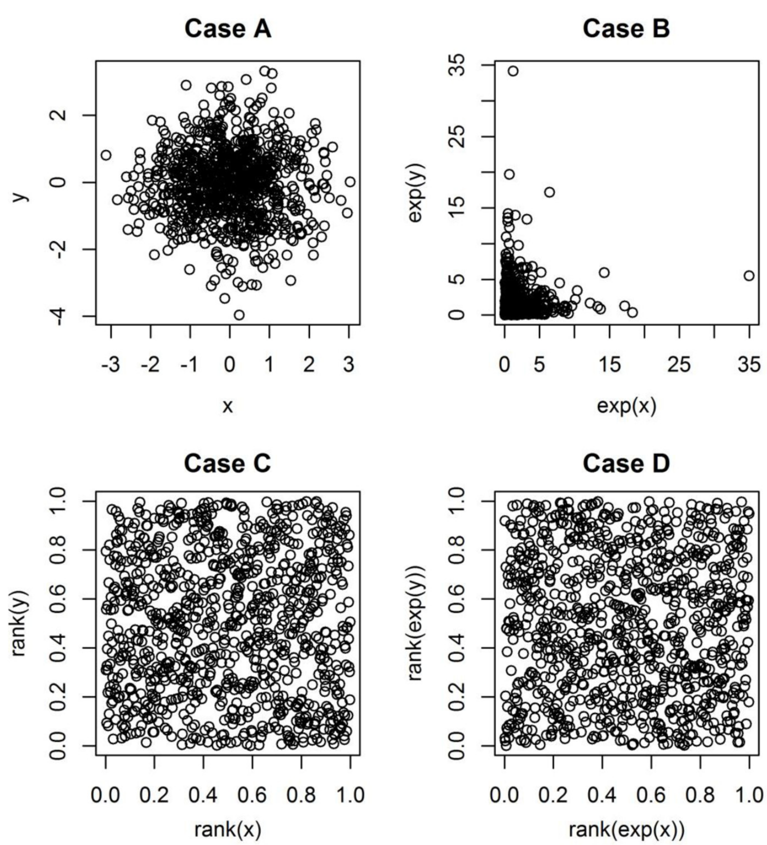
BACKGROUND

The coefficient of linear correlation is a dependence measure that plays a central role in the financial and actuarial theory. However, outside the context of the elliptical distributions, it suffers from subtle pitfalls. Among its important drawbacks, discussed in Embrechts, McNeil, & Straumann (2002) and McNeil et al. (2005), is that the coefficient of linear correlation is a scalar measure of linear dependence. Therefore, it cannot describe successfully the whole dependence structure. Furthermore, it is defined only for finite variances. This means that not all of its values are attainable and that problems may be faced when the explored random variables have heavy-tailed distributions. In addition, the coefficient remains invariant only under strictly increasing linear transformations of the random variables. Strictly increasing non-linear transformations, for example exponential, alter the dependence structure (Figure 1, Case B). This statement is explained by the fact that the coefficient of linear correlation does not only incorporate information about the dependence between the participating two random variables, but also about their marginal behavior. Rank correlation copula-based measures of dependence, such as Spearman's rho and Kendall's tau, provide a solution to the outlined problem, because they do not depend on the the marginal behavior of the random variables, and remain invariant under their strictly monotonic (linear and non-linear) transformations. Figure 1, Case D illustrates this point by demonstrating that if one works with the

Figure 1. Linear vs. rank correlation. Case A: A scatter plot of 900 generated observations of two random variables x and y that belong to a Student's t distribution with 25 degrees of freedom. Case B: A scatter plot of a non-linear transformation of x and y from Case A. Case C: A scatter plot of the ranks of x and y from Case A. Case D: A scatter plot of the ranks of the observations from case B.



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