

Neural Networks and Equilibria, Synchronization, and Time Lags

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INTRODUCTION

All neural networks, both natural and artificial, are characterized by two kinds of dynamics. The first one is concerned with what we would call “learning dynamics”, in fact the sequential (discrete time) dynamics of the choice of synaptic weights. The second one is the intrinsic dynamics of the neural network viewed as a **dynamical system** after the weights have been established *via* learning. Regarding the second dynamics, the emergent computational capabilities of a **recurrent neural network** can be achieved provided it has **many equilibria**. The network task is achieved provided it approaches these equilibria. But the **dynamical system** has a dynamics induced *a posteriori* by the learning process that had established the synaptic weights. It is not compulsory that this *a posteriori* dynamics should have the required properties, hence they have to be checked separately.

The standard stability properties (Lyapunov, asymptotic and exponential stability) are defined for a single equilibrium. Their counterpart for **several equilibria** are: *mutability, global asymptotics, gradient behavior*. For the definitions of these general concepts the reader is sent to Gelig *et. al.*, (1978), Leonov *et. al.*, (1992).

In the last decades, the number of **recurrent neural networks**’ applications increased, they being designed for classification, identification and complex image, visual and spatio-temporal processing in fields as engineering, chemistry, biology and medicine (see, for instance: Fortuna *et. al.*, 2001; Fink, 2004; Atencia *et. al.*, 2004; Iwahori *et. al.*, 2005; Maurer *et. al.*, 2005; Guirguis & Ghoneimy, 2007). All these applications are mainly based on the existence of **several equilibria** for such networks, requiring them the “good behavior” properties above discussed.

Another aspect of the qualitative analysis is the so-called **synchronization** problem, when an external

stimulus, in most cases periodic or almost periodic has to be tracked (Gelig, 1982; Danciu, 2002). This problem is, from the mathematical point of view, nothing more but existence, uniqueness and global stability of forced oscillations.

In the last decades the neural networks dynamics models have been modified once more by introducing the transmission **delays**. The standard model of a Hopfield-type network with delay as considered in (Gopalsamy & He, 1994) is

$$\frac{du_i}{dt} = -a_i u_i(t) + \sum_{j=1}^n w_{ij} g_j(u_j(t - \tau_{ij})) + I_i \quad i = \overline{1, n} \quad (1)$$

The present paper aims to a general presentation, with both research and educational purposes, of the three topics mentioned previously.

BACKGROUND

Dynamical systems with **several equilibria** occur in such fields of science and technology as electrical machines, chemical reactions, economics, biology and, last but not least, neural networks.

For systems with **several equilibria** the usual local concepts of stability are not sufficient for an adequate description. The so-called “global phase portrait” may contain both stable and unstable equilibria: each of them may be characterized separately since stability is a local concept dealing with a specific trajectory. But global concepts are also required for a better system description and this is particularly true for the case of the neural networks. Indeed, the neural networks may be viewed as interconnections of simple computing elements whose computational capability is increased by interconnection (“emergent collective capacities”

– to cite Hopfield). This is due to the nonlinear characteristics leading to the existence of several stable equilibria. The network achieves its computing goal if no self-sustained oscillations are present and it always achieves some steady-state (equilibrium) among a finite (while large) number of such states.

This behavior is most suitably described by the concepts arising from the papers of Kalman (1957) and Moser (1967). The last of them relies on the following remark concerning the rather general nonlinear autonomous system

$$\dot{x} = -f(x), \quad x \in \mathbb{R}^n \quad (2)$$

where $f(x) = \text{grad } G(x)$ and $G : \mathbb{R}^n \rightarrow \mathbb{R}$ is such that the number of its critical points is finite and is radially unbounded i.e. $\lim_{|x| \rightarrow \infty} G(x) = \infty$. Under these assumptions any solution of (2) approaches asymptotically one of the equilibria (which is also a critical point of G – where its gradient, i.e. f vanishes). Obviously the best limit behavior of a neural network would be like this – naturally called **gradient like behavior**. Nevertheless there are other properties that are also important while weaker; in the following we shall discuss some of them.

The mathematical object will be in the following the system of ordinary differential equations

$$\dot{x} = f(x, t) \quad (3)$$

and we shall first define some basic notions.

Definition 1 a) Any constant solution of (3) is called **equilibrium**; the set of equilibria E is called **stationary set**. b) A solution of (3) is called **convergent** if it approaches asymptotically some equilibrium:

$$\lim_{t \rightarrow \infty} x(t) = c \in E \quad (4)$$

A solution is called **quasi-convergent** if it approaches asymptotically the stationary set:

$$\lim_{t \rightarrow \infty} d(x(t), E) = 0, \quad (5)$$

with $d(z, M)$ being the distance (in the usual sense) from the point z to the set M .

c) System (3) is called **monostable (strictly mutable)** if every bounded solution is convergent (in the above

sense); it is called **quasi-monostable** if every bounded solution is quasi-convergent.

d) System (3) is called **gradient-like** if every solution is convergent; it is called **quasi-gradient-like (has global asymptotics)** if every solution is quasi-convergent.

Remark that *convergence* is a solution property while *monostability* and *gradient* property are associated to systems. For autonomous (time invariant) systems of the form (2) the following Lyapunov type results are available.

Lemma 1 Consider system (2) and assume existence of a continuous function $V : \mathbb{R}^n \rightarrow \mathbb{R}$ that is nonincreasing along any of its solutions. If, additionally, a bounded on \mathbb{R}^+ solution $x(t)$ for which there exists some $\tau > 0$ such that $V(x(\tau)) = V(x(0))$ is an equilibrium, then the system is quasi-monostable.

Lemma 2 If the assumptions of Lemma 1 hold and, additionally, $V(x)$ is radially unbounded then system (2) is quasi-gradient like.

Lemma 3 If the assumptions of Lemma 2 hold and the set E is discrete (i.e. it consists of isolated points only) then system (2) is gradient-like.

DYNAMICS ISSUES OF RECURRENT NEURAL NETWORKS

Neural Networks as Systems with Several Equilibria

It has been already mentioned that the emergent computational capacities of the neural networks are ensured by: a) nonlinear behavior of the neural cells; b) their connectivity. These two properties define the neural networks as **dynamical systems with many equilibria** whose performance depends on the (high) number of these equilibria and on the **gradient like property** of the network.

On the other hand, the standard **recurrent neural networks** (Bidirectional Associative Memory (Kosko, 1988), Hopfield (1982), cellular (Chua & Yang, 1988), Cohen-Grossberg (1983)), which contain internal feedback loops - having thus the propensity for instability, possess some “natural”, i.e. associated in a natural way,

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