Nelder–Mead Evolutionary Hybrid Algorithms

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INTRODUCTION

Real world optimization problems are often too complex to be solved through analytic means. Evolutionary algorithms are a class of algorithms that borrow paradigms from nature to address them. These are stochastic methods of optimization that maintain a population of individual solutions, which correspond to points in the search space of the problem. These algorithms have been immensely popular as they are derivative-free techniques, are not as prone to getting trapped in local minima, and can be tailored specifically to suit any given problem. The performance of evolutionary algorithms can be improved further by adding a local search component to them. The Nelder-Mead simplex algorithm (Nelder & Mead, 1965) is a simple local search algorithm that has been routinely applied to improve the search process in evolutionary algorithms, and such a strategy has met with great success.

In this article, we provide an overview of the various strategies that have been adopted to hybridize two well-known evolutionary algorithms - genetic algorithms (GA) and particle swarm optimization (PSO).

BACKGROUND

Arguably, GAs are one of the most of all common population based approaches for optimization. The population of candidate solutions that these algorithms maintain in each generation are called chromosomes. GAs carry out the Darwinian operators of selection, mutation, and recombination, on these chromosomes, to perform their search (Mitchell, 1998). Each generation is improved by removing the poorer solutions from the population, while retaining the better ones, based on a fitness measure. This process is called selection. Following selection, a method of recombining solutions called crossover is applied. Here two (or more) parent solutions from the current generation are picked randomly for producing offspring to populate the next generation of solutions. The offspring chromosomes are then probabilistically subject to mutation, which is carried out by the addition of small random perturbations.

PSO is a more recent approach for optimization (Kennedy & Eberhart, 2001). Being modelled after the social behavior of organisms such as a flock of birds in flight or a school of fish swimming, it is considered an evolutionary algorithm only in a loose sense. Each solution within the population is called a particle in PSO. Each such particle's position in the search space is constantly updated within each generation, by the addition of the particle's velocity to it. The velocity of a particle is then adjusted towards the best position encountered in the particle's own history (individual best), as well as the best position in the current iteration (global best).

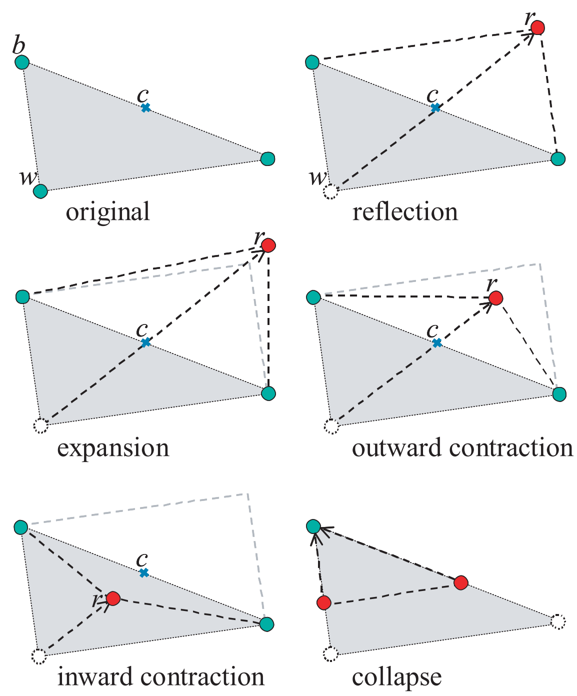
Since evolutionary algorithms use a population of individuals and randomized variational operators, they are adept at performing exploratory searches over their search spaces. However, when the aim is to produce outputs within reasonable time limits, it is important to balance this exploration with better exploitation of smaller-scale features in the fitness landscape. In the latter context, local search algorithms enable single solutions to be improved using local information (e.g., directional trends in fitness around each solution) and take the solution towards the closest maximum fitness. Hybrid algorithms that combine the advantages of exploration and exploitation comprise of a distinct area of evolutionary computation research that have been variously called as Lamarckian or memetic approaches, of which Nelder-Mead hybrids are a significant chunk.

NELDER-MEAD SIMPLEX BASED HYBRIDIZATION

The Nelder-Mead Downhill Simplex Algorithm

The Nelder-Mead simplex algorithm is a derivative-free local search technique that is capable of moving a cluster

Figure 1. Various operations in the Nelder-Mead simplex routine



of solutions in the gradient direction and which, as per current research, can be very effectively combined with GA and PSO approaches. These hybrid evolutionary algorithms have been shown to be very successful in continuous optimization problems.

The Nelder-Mead smplex method makes use of a construct called a simplex (see Figure 1.). When the search space is n-dimensional, the simplex consists of n+1 solutions, s_i, $i = \{1, 2, \ldots, n+1\}$, that are usually closely spaced. As shown in the top left of Figure 1., in a two-dimensional search plane, a simplex is a triangle. The fitness of each solution is considered in each step of the Nelder-Mead method, and the worst solution w is identified. The centroid, c, of the remaining n points

$$c = \frac{1}{n}\sum s_i,$$

is computed and the reflection of w along it determined. This reflection yields a new solution r that replaces w, in the next step, as shown in the top right of Figure 1. If the solution r produced by this reflection has a higher fitness than any other solution in the simplex, the simplex is further expanded along the direction of r, as shown in the middle left of the figure. On the other hand, if r has a low fitness compared to the others, the simplex is contracted. Contraction can be either outward or inward depending upon whether r is better or worse than w. The contraction operations are shown in the middle right and bottom left of the figure. If neither contraction improves the worst solution in the simplex, the best point in the simplex is computed, and a collapse is then carried out, and all the points of the simplex are moved a little closer towards the best one, as shown in the bottom right of the same figure.

The approaches taken to incorporate a simplex-based local search routine within the broad framework of a genetic algorithm fall under four different schemes that are shown in Figure 2. These are as follows:

Two-Phase Hybridization

This is the simplest of all approaches and has been applied to GAs (Chelouah & Siarry, 2000, Chelouah & Siarry, 2003, Robin, Orzati, Moreno, Homan & Bachtold, 2003). In the first phase in this scheme, a GA is applied to the optimization problem to explore the entire search space until one or more good solutions