MREM, Discrete Recurrent Network for Optimization

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INTRODUCTION

Since McCulloch and Pitts' seminal work (McCulloch & Pitts, 1943), several models of discrete neural networks have been proposed, many of them presenting the ability of assigning a discrete value (other than unipolar or bipolar) to the output of a single neuron. These models have focused on a wide variety of applications. One of the most important models was developed by J. Hopfield in (Hopfield, 1982), which has been successfully applied in fields such as pattern and image recognition and reconstruction (Sun et al., 1995), design of analog digital circuits (Tank & Hopfield, 1986), and, above all, in combinatorial optimization (Hopfield & Tank, 1985) (Takefuji, 1992) (Takefuji & Wang, 1996), among others.

The purpose of this work is to review some applications of multivalued neural models to combinatorial optimization problems, focusing specifically on the neural model MREM, since it includes many of the multivalued models in the specialized literature.

BACKGROUND

In Hopfield and Tank's pioneering work (Hopfield & Tank, 1985), neural networks were applied for the first time to solve combinatorial optimization problems, concretely the well-known travelling salesman problem. They developed two types of networks, discrete and continuous, although the latter has been mostly chosen to solve optimization problems, adducing that it helps to escape more easily from local optima. Since then, the search for better neural algorithms, to face the diverse problems of combinatorial optimization (many of them

belonging to the class of NPcomplete problems), has been the objective of researchers in this field.

This method of optimization consists of minimizing an energy function, whose parameters and constraints are obtained by means of identification with the objective function of the optimization problem. In this case, the energy function has the form:

$$E(\boldsymbol{S}) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{i,j}s_i s_j + \sum_{i=1}^{N}\theta_i s_i$$

where N is the number of neurons of the network, $w_{i,j}$ is the synaptic weight between neurons j and i, and θ_i is the threshold or bias of the neuron i.

In the discrete version of Hopfield's model, component s_i of the state vector $\boldsymbol{S} = (s_1,...,s_N)$ can take values in $\mathcal{M} = \{-1,1\}$ (constituting the bipolar model) or in $\mathcal{M} = \{0,1\}$ (unipolar model). In the continuous version, $\mathcal{M} = [-1,1]$ or $\mathcal{M} = [0,1]$. This continuous version, although it has been traditionally the most used for optimization problems, presents certain inconveniences:

- Certain special mechanisms, maybe in form of constraints, should be contributed in order to get that, in the final state of the network, all the components of state vector $\boldsymbol{S}$ belong to $\{-1, 1\}$ or $\{0,1\}$.
- The traditional dynamics used in this model, implemented in a digital computer, does not guarantee the decrease of the energy function in every iteration, so it is not ensured that the final state is a minimum of the energy function (Galán-Marín, 2000).

However, the biggest problem of this model (the discrete as well as the continuous one) is the possibility to converge to a non feasible state, or to a local (not global) minimum. Wilson and Pawley (1988) demonstrated, through massive simulations, that, for the travelling salesman problem of 10 cities, only 8% of the solutions were feasible, and most not good. Moreover, this proportion got worse when problem size was increased.

After this, many works were focused on improving Hopfield's network:

- By modifying the energy function (Xu & Tsai, 1991).
- By adjusting the numerous parameters present in the network, as in (Lai & Coghill, 1988).
- By using stochastic techniques in the dynamics of the network (Kirkpatrick et al., 1983) (Aarts & Korst, 1988).

Particularly, researchers tried to improve the efficiency of Hopfield's network for the travelling salesman problem, achieving acceptable results, but inferior to Operations Research techniques (Takahashi, 1997). The reason for these disappointing results is that the linear formulation used by these techniques is a great advantage in comparison with neural networks, which unavoidably use a quadratic energy function, impeding the use of subpaths deletion techniques (Smith, 1996), and provoking the appearance of a bigger number of local minima.

Another research line was devoted to the improvement of Hopfieldtype recurrent networks, and their application to diverse problems of optimization, in which some results proved to be better than those obtained by traditional Operations Research techniques (Smith & Krishnamoorthy, 1998). Takefuji's work (Takefuji, 1992) (Lee et al., 1992)(Takefuji & Wang, 1996), with a great number of publications in international media, must be highlighted. Their results have been overcome by the OCHOM model (GalánMarín & MuñozPérez, 2001).

MULTIVALUED DISCRETE RECURRENT MODEL. APPLICATION TO COMBINATORIAL OPTIMIZATION PROBLEMS

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A new generalization of Hopfield's model arises in the works (MéridaCasermeiro, 2000) (MéridaCasermeiro et al., 2001), where the MREM (Multivalued REcurrent Model) model is presented.

The Neural MREM Model

This model presents two essential features that make it very versatile and that increase its applicability:

- The output of each neuron, s_i, is a value of the set $\mathcal{M} = \{m_1, m_2, \ldots, m_L\}$, which is not necessarily numeric.
- The concept of similarity function f between neuron outputs is introduced. $f(x,y)$ represents the similarity between neuron states x and y.

This way, the energy function of this model is as follows:

$$E(\boldsymbol{S}) = -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} w_{i,j} f(s_i, s_j) + \sum_{i=1}^{N} \theta_i(s_i)$$

where $\theta_i : \mathcal{M} \to \mathbb{R}$ is a generalization of the thresholds of each neuron.

The features mentioned above make that in this model certain optimization problems (as the travelling salesman problem), have a better representation than in the unipolar or bipolar Hopfield's models, and their successors.

It is clear that MREM includes Hopfield's models (with outputs in $\mathcal{M} = \{-1,1\}$ or in $\mathcal{M} = \{0,1\}$) if we consider the similarity function given by the product $f(a,b) = ab$. Other multivalued models, like MAREN or SOAR (Erdem & Ozturk, 1996) (Ozturk & Abut, 1997), are also generalized by MREM.

The dynamics for this network is chosen according to the problem to be tackled.

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