

Modal Logics for Reasoning about Multiagent Systems

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INTRODUCTION

It becomes evident in recent years a surge of interest to applications of modal logics for specification and validation of complex systems. It holds in particular for combined logics of knowledge, time and actions for reasoning about multiagent systems (Dixon, Nalon & Fisher, 2004; Fagin, Halpern, Moses & Vardi, 1995; Halpern & Vardi, 1986; Halpern, van der Meyden & Vardi, 2004; van der Hoek & Wooldridge, 2002; Lomuscio, & Penczek, W., 2003; van der Meyden & Shilov, 1999; Shilov, Garanina & Choe, 2006; Wooldridge, 2002). In the next paragraph we explain what are logics of knowledge, time and actions from a viewpoint of mathematicians and philosophers. It provides us a historic perspective and a scientific context for these logics.

For mathematicians and philosophers logics of actions, time, and knowledge can be introduced in few sentences. A logic of actions (ex., Elementary Propositional Dynamic Logic (Harel, Kozen & Tiuryn, 2000)) is a polymodal variant of a basic modal logic **K** (Bull & Segerberg, 2001) to be interpreted over arbitrary Kripke models. A logic of time (ex., Linear Temporal Logic (Emerson, 1990)) is a modal logic with a number of modalities that correspond to “next time”, “always”, “sometimes”, and “until” to be interpreted in Kripke models over partial orders (discrete linear orders for LTL in particular). Finally, a logic of knowledge or epistemic logic (ex., Propositional Logic of Knowledge (Fagin, Halpern, Moses & Vardi, 1995; Rescher, 2005)) is a polymodal variant of another basic modal logic **S5** (Bull & Segerberg, 2001) to be interpreted over Kripke models where all binary relations are equivalences.

BACKGROUND: MODAL LOGICS

All **modal logics** are languages that are characterized by syntax and semantics. Let us define below a very simple modal logic in this way. This logic is called **Elementary Propositional Dynamic Logic (EPDL)**.

Let *true*, *false* be Boolean constants, *Prp* and *Rel* be disjoint sets of propositional and relational variable respectively. The syntax of the classical propositional logic consists of formulas which are constructed from propositional variables and Boolean connectives “¬” (negation), “&” (conjunction), “∨” (disjunction), “→” (implication), and “↔” (equivalence) in accordance with the standard rules. EPDL has additional formula constructors, modalities, which are associated with relational variables: if *r* is a relational variable and φ is a formula of EPDL then

- $([r]\varphi)$ is a formula which is read as “box *r*- φ ” or “after *r* always φ ”;
- $(\langle r \rangle \varphi)$ is a formula which is read as “diamond *r*- φ ” or “after *r* sometimes φ ”.

The semantics of EPDL is defined in models, which are called **labeled transition systems** by computer scientists and **Kripke models**¹ by mathematicians and philosophers. A model *M* is a pair (D, I) where the domain (or the universe) $D \neq \emptyset$ is a set, while the interpretation *I* is a pair of mappings (P, R) . Elements of the domain *D* are called states by computer scientists and worlds by mathematicians and philosophers. The interpretation maps propositional variables to sets of states $P: Prp \rightarrow 2^D$ and relational variables to binary relations on states $R: Rel \rightarrow 2^{D \times D}$. We write $I(p)$ and

$I(r)$ instead of $P(p)$ and $R(r)$ whenever it is implicit that p and r are propositional and relational variables respectively.

Every model $M = (D, I)$ can be viewed as a directed graph with nodes and edges labeled by propositional and action variables respectively. Its nodes are states of D . A node $s \in D$ is marked by a propositional variable $p \in Prp$ iff $s \in I(p)$. A pair of nodes $(s_1, s_2) \in D \times D$ is an edge of the graph iff $(s_1, s_2) \in I(r)$ for some relational variable $r \in Rel$; in this case the edge (s_1, s_2) is marked by this relational variable r . Conversely, a graph with nodes and edges labeled by propositional and relational variables respectively can be considered as a model.

For every model $M = (D, I)$ the **entailment** (validity, satisfiability) **relation** \models_M between states and formulas can be defined by induction on formula structure:

- for every state $s \models_M true$ and not $s \models_M false$;
- for any state s and propositional variable p , $s \models_M p$ iff $s \in I(p)$;
- for any state s and formula φ , $s \models_M (\neg\varphi)$ iff it is not the case $s \models_M \varphi$;
- for any state s and formulas φ and ψ ,
 $s \models_M (\varphi \ \& \ \psi)$ iff $s \models_M \varphi$ and $s \models_M \psi$;
 $s \models_M (\varphi \ \vee \ \psi)$ iff $s \models_M \varphi$ or $s \models_M \psi$;
- for any state s , relational variable r , and formula φ ,
 $s \models_M ([r]\varphi)$ iff $(s, s') \in I(r)$ and $s' \models_M \varphi$ for every state s' ;
 $s \models_M (\langle r \rangle \varphi)$ iff $(s, s') \in I(r)$ and $s' \models_M \varphi$ for some state s' .

Semantics of the above kind is called **possible worlds semantics**.

Let us explain EPDL pragmatics by the following puzzle example.

Alice and Bob play the Number Game. Positions in the game are integers in [1..109]. An initial position is a random number. Alice and Bob make alternating moves: Alice, Bob, Alice, Bob, etc. Available moves are same for both: if a current position is $n \in [1..99]$ then $(n+1)$ and $(n+10)$ are possible next positions. A player wins the game iff the opponent is the first to enter [100..109]. Problem: Find all initial positions where Alice has a winning strategy.

Kripke model for the game is quite obvious:

- States correspond to game positions, i.e. integers in [1..109].
- Propositional variable *fail* is interpreted by [100..109].
- Relational variable *move* is interpreted by possible moves.

Formula $\neg fail \ \& \ \langle move \rangle (\neg fail \ \& \ [move] fail)$ is valid in those states where the game is not lost, there exists a move after which the game is not lost, and then all possible moves always lead to a loss in the game. Hence this EPDL formula is valid in those states where Alice has a 1-round winning strategy against Bob.

COMBINING KNOWLEDGE, ACTIONS AND TIME

Logic of Knowledge

Logics of knowledge are also known as **epistemic logics**. One of the simplest epistemic logic is **Propositional Logic of Knowledge for $n > 0$ agents (PLK_n)** (Fagin, Halpern, Moses & Vardi, 1995). A special terminology, notation and **Kripke models** are used in this framework. A set of relational symbols *Rel* in PLK_n consists of natural numbers [1..n] representing names of agents. Notation for modalities is: if $i \in [1..n]$ and φ is a formula, then $(Ki \ \varphi)$ and $(Si \ \varphi)$ are used instead of $([i] \ \varphi)$ and $(\langle i \rangle \ \varphi)$. These formulas are read as “(an agent) i knows φ ” and “(an agent) i can suppose φ ”. For every agent $i \in [1..n]$ in every model $M = (D, I)$, interpretation $I(i)$ is an “**indistinguishability relation**”, i.e. an equivalence relation² between states that the agent i can not distinguish. Every model M , where all agents are interpreted in this way, is denoted as $(D, \sim_1 \ \dots \ \sim_n, I)$ with explicit $I(I) = \sim_1, \dots, I(n) = \sim_n$ instead of brief standard notation (D, I) . An agent knows some “fact” φ in a state s of a model M , if the fact is valid in every state s' of this model that the agent can not distinguish from s :

- $s \models_M (K_i \ \varphi)$ iff $s' \models_M \varphi$ for every state $s' \sim_i s$.

Similarly, an agent can suppose a “fact” φ in a state s of a model M , if the fact is valid in some state s' of this model that the agent can not distinguish from s :

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