

Fuzzy Rule Interpolation

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INTRODUCTION

The “fuzzy dot” (or fuzzy relation) representation of fuzzy rules in fuzzy rule based systems, in case of classical fuzzy reasoning methods (e.g. the Zadeh-Mamdani-Larsen Compositional Rule of Inference (CRI) (Zadeh, 1973) (Mamdani, 1975) (Larsen, 1980) or the Takagi - Sugeno fuzzy inference (Sugeno, 1985) (Takagi & Sugeno, 1985)), are assuming the completeness of the fuzzy rule base. If there are some rules missing i.e. the rule base is “sparse”, observations may exist which hit no rule in the rule base and therefore no conclusion can be obtained. One way of handling the “fuzzy dot” knowledge representation in case of sparse fuzzy rule bases is the application of the Fuzzy Rule Interpolation (FRI) methods, where the derivable rules are deliberately missing. Since FRI methods can provide reasonable (interpolated) conclusions even if none of the existing rules fires under the current observation. From the beginning of 1990s numerous FRI methods have been proposed. The main goal of this article is to give a brief but comprehensive introduction to the existing FRI methods.

BACKGROUND

Since the classical fuzzy reasoning methods (e.g. the Zadeh-Mamdani-Larsen CRI) are demanding complete rule bases, the classical rule base construction claims a special care of filling all the possible rules. In case if the rule base is “sparse” (some rules are missing), observations may exist which hit no rule and hence no conclusion can be obtained. In many application areas of fuzzy control structures, the accidental lack of conclusion is hard to explain, or meaningless (e.g. in steering control of a vehicle). This case one obvious solution could be to keep the last real conclusion instead of the missing one, but applying historical data automatically to fill undeliberately missing rules could cause unpredictable side effects. Another solution for the same problem is the application of the fuzzy rule

interpolation (FRI) methods, where the derivable rules are deliberately missing. The rule base of an FRI controller is not necessarily complete, since FRI methods can provide reasonable (interpolated) conclusions even if none of the existing rules fires under the current observation. It could contain the most significant fuzzy rules only, without risking the chance of having no conclusion for some of the observations. On the other hand most of the FRI methods are sharing the burden of high computational demand, e.g. the task of searching for the two closest surrounding rules to the observation, and calculating the conclusion at least in some characteristic α -cuts. Moreover in some methods the interpretability of the fuzzy conclusion gained is also not straightforward (Kóczy & Kovács, 1993). There have been a lot of efforts to rectify the interpretability of the interpolated fuzzy conclusion (Tikk & Baranyi, 2000). In (Baranyi, Kóczy & Gedeon, 2004) Baranyi *et al.* give a comprehensive overview of the recent existing FRI methods. Beyond these problems, some of the FRI methods are originally defined for one dimensional input space, and need special extension for the multidimensional case (e.g. (Jenei, 2001), (Jenei, Klement & Konzel, 2002)). In (Wong, Tikk, Gedeon & Kóczy, 2005) Wong *et al.* gave a comparative overview of the recent multidimensional input space capable FRI methods. In (Jenei, 2001) Jenei introduced a way for axiomatic treatment of the FRI methods. In (Perfilieva, 2004) Perfilieva studies the solvability of fuzzy relation equations as the solvability of interpolating and approximating fuzzy functions with respect to a given set of fuzzy rules (e.g. fuzzy data as ordered pairs of fuzzy sets). The high computational demand, mainly the search for the two closest surrounding rules to an arbitrary observation in the multidimensional antecedent space turns many of these methods hardly suitable for real-time applications. Some FRI methods, e.g. the method introduced by Jenei *et al.* in (Jenei, Klement & Konzel, 2002), eliminate the search for the two closest surrounding rules by taking all the rules into consideration, and therefore speeding up the reasoning process. On the other hand, keeping the goal of con-

structuring fuzzy conclusion, and not simply speeding up the reasoning, they still require some additional (or repeated) computational steps for the elements of the level set (or at least for some relevant α levels). An application oriented aspect of the FRI emerges in (Kovács, 2006), where for the sake of reasoning speed and direct real-time applicability, the fuzziness of fuzzy partitions replaced by the concept of Vague Environment (Klawonn, 1994).

In the followings, the brief structure of several FRI methods will be introduced in more details.

FUZZY RULE INTERPOLATION METHODS

One of the first FRI techniques was published by Kóczy and Hirota (Kóczy & Hirota, 1991). It is usually referred as KH method. It is applicable to convex and normal fuzzy (CNF) sets in single input and single output (SISO) systems. The KH method takes into consideration only the two closest surrounding (flanking) rules to the observation. It determines the conclusion by its α -cuts in such a way that the ratio of distances between the conclusion and the consequents should be identical with the ratio of distances between the observation and the antecedents for all important α -cuts. The applied formula:

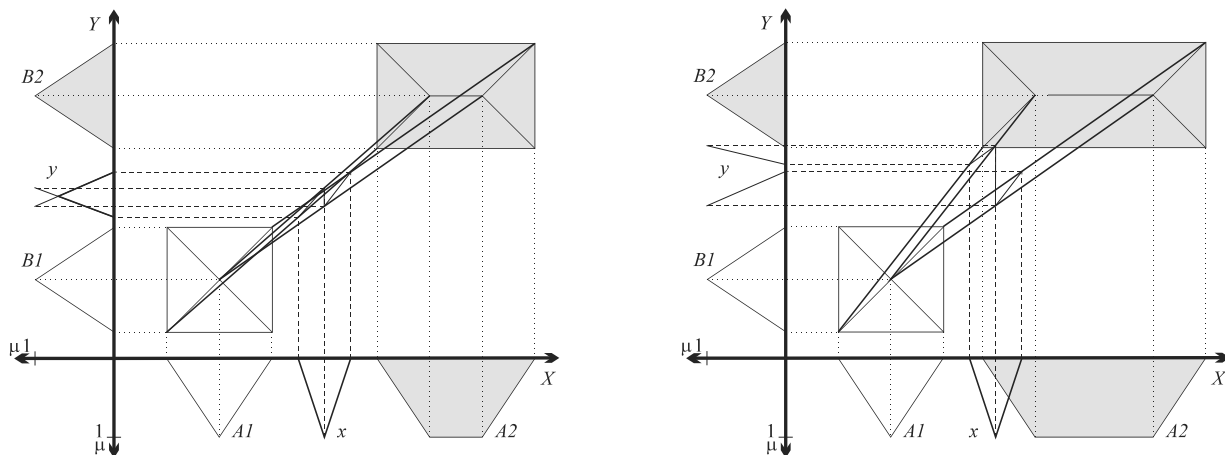
$$d(A^*, A_1) : d(A^*, A_2) = d(B^*, B_1) : d(B^*, B_2),$$

can be solved for the required conclusion B^* for relevant α -cuts after decomposition. Where $A_1 \rightarrow B_1$ and $A_2 \rightarrow B_2$ are the two flanking rules of the observation A^* and $d: F(X) \times F(X) \rightarrow R$ is a distance function of fuzzy sets (in case of the KH method it was calculated as the distance of the lower and upper end points of the α -cuts) (see e.g. on Fig. 1.).

It is shown in, e.g. in (Kóczy & Kovács, 1993), (Kóczy & Kovács, 1994) that the conclusion of the KH method is not always directly interpretable as fuzzy set (see e.g. on Fig. 1.). This drawback motivated many alternative solutions. The first modification was proposed by Vass, Kalmár and Kóczy (Vass, Kalmár & Kóczy, 1992) (referred as VKK method), where the conclusion is computed based on the distance of the centre points and the widths of the α -cuts, instead of their lower and upper end point distances. The VKK method extends the applicability of the KH method, but it was still strongly depends on the membership shape of the fuzzy sets (e.g. it was unable to handle singleton antecedent sets, as the width of the antecedent's support must not be zero).

In spite of the known restrictions, the KH method is still popular because of its simplicity. Subsequently it was generalized in several ways. Among them the stabilized KH interpolator was emerged, as it was proved

Figure 1. KH method for two SISO rules: $A_1 \rightarrow B_1$ and $A_2 \rightarrow B_2$, conclusion y of the observation x



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