Fuzzy Control Systems: An Introduction

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INTRODUCTION

Fuzzy control systems are developed based on fuzzy set theory, attributed to Lotfi A. Zadeh (Zadeh, 1965, 1973), which extends the classical set theory with memberships of its elements described by the classical characteristic function (either "is" or "is not" a member of the set), to allow for partial membership described by a membership function (both "is" and "is not" a member of the set at the same time, with a certain degree of belonging to the set). Thus, fuzzy set theory has great capabilities and flexibilities in solving many real-world problems which classical set theory does not intend or fails to handle.

Fuzzy set theory was applied to control systems theory and engineering almost immediately after its birth. Advances in modern computer technology continuously backs up the fuzzy framework for coping with engineering systems of a broad spectrum, including many control systems that are too complex or too imprecise to tackle by conventional control theories and techniques.

BACKGROUND: FUZZY CONTROL SYSTEMS

The main signature of fuzzy logic technology is its ability of suggesting an approximate solution to an imprecisely formulated problem. From this point of view, fuzzy logic is closer to human reasoning than the classical logic, where the latter attempts to precisely formulate and exactly solve a mathematical or technical problem if ever possible.

Motivations for Fuzzy Control Systems Theory

Conventional control systems theory, developed based on classical mathematics and the two-valued logic, is relatively mature and complete. This theory has its solid foundation built on classical mathematics, electrical engineering, and computer technology. It can provide rigorous analysis and often perfect solutions when a system is precisely defined mathematically. Within this framework, some relatively advanced control techniques such as adaptive, robust and nonlinear control theories have gained rapid development in the last three decades.

However, conventional control theory is quite limited in modeling and controlling complex dynamical systems, particularly ill-formulated and partially-described physical systems. Fuzzy logic control theory, on the contrary, has shown potential in these kinds of non-traditional applications. Fuzzy logic technology allows the designers to build controllers even when their understanding of the system is still in a vague, incomplete, and developing phase, and such situations are quite common in industrial control practice.

General Structure of Fuzzy Control Systems

Just like other mathematical tools, fuzzy logic, fuzzy set theory, fuzzy modeling, fuzzy control methods, etc., have been developed for solving practical problems. In control systems theory, if the fuzzy interpretation of a real-world problem is correct and if fuzzy theory is developed appropriately, then fuzzy controllers can be suitably designed and they work quite well to their advantages. The entire process is then returned to the original real-world setting, to accomplish the desired system automation. This is the so-called "fuzzification—fuzzy operation—defuzzification" routine in fuzzy control design. The key step—fuzzy operation—is executed by a logical rule base consisting of some IF-THEN rules established by using fuzzy logic and human knowledge (Chen & Pham, 1999, 2006; Drianker, Hellendoorn & Reinfrank, 1993; Passino & Yurkovich, 1998; Tanaka, 1996; Tanaka & wang, 1999; Wang, 1994; Ying, 2000).

Fuzzification

Fuzzy set theory allows partial membership of an element with respect to a set: an element can partially belong to a set and meanwhile partially not belong to the same set. For example, an element, x, belonging to the set, X, IS specified by a (normalized) membership function, $\mu_X : X \rightarrow [0,1]$. There are two extreme cases: $\mu_X(x) = 0$ means $x \notin X$ and $\mu_X(x) = 1$ means $x \in X$ in the classical sense. But $\mu_X(x) = 0.2$ means x belongs to X only with grade 0.2, or equivalently, x does not belong to X with grade 0.8. Moreover, an element can have more than one membership value at the same time, such as $\mu_X(x) = 0.2$ and $\mu_X(x) = 0.6$, and they need not be summed up to one. The entire setting depends on how large the set X is (or the sets X and Y are) for the associate members, and what kind of shape a membership function should have in order to make sense of the real problem at hand. A set, X, along with a membership function defined on it, $\mu_X(\cdot)$, is called a *fuzzy set* and is denoted (X, μ_X). More examples of fuzzy sets can be seen below, as the discussion continues. This process of transforming a crisp value of an element (say $x = 0.3$) to a fuzzy set (say $x = 0.3 \in X = [0,1]$ with $\mu_X(x) = 0.2$) is called *fuzzification*.

Given a set of real numbers, $X = [-1,1]$, a point $x \in X$ assumes a real value, say $x = 0.3$. This is a crisp number without fuzziness. However, if a membership function $\mu_X(\cdot)$ is introduced to associate with the set X, then (X, μ_X) becomes a fuzzy set, and the (same) point $x = 0.3$ has a membership grade quantified by $\mu_X(\cdot)$ (for instance, $\mu_X(x) = 0.9$). As a result, x has not one but two values associated with the point: $x = 0.3$ and $\mu_X(x) = 0.9$. In this sense, x is said to have been *fuzzified*. For convenience, instead of saying that "x is in the set X with a membership value $\mu_X(x)$," in common practice it is usually said "x is ," while one should keep in mind that there is always a well-defined membership function associated with the set X. If a member, x, belongs to two fuzzy sets, one says "x is X_1 AND x is X_2," and so on. Here, the relation AND needs a logical operation to perform. As a result, this statement eventually yields only one membership value for the element x, denoted by $\mu_{X_1 \times X_2}(x)$. There are several logical operations to implement the logical AND; they are quite different but all valid within their individual logical system. A commonly used one is $\mu_{X_1 \times X_2}(x) = \min \{\mu_{X_1}(x), \mu_{X_2}(x)\}$.

Fuzzy Logic Rule Base

The majority of fuzzy logic control systems are knowledge-based systems. This means that either their fuzzy models or their fuzzy logic controllers are described by fuzzy logic IF-THEN rules. These rules have to be established based on human expert's knowledge about the system, the controller, and the performance specifications, etc., and they must be implemented by performing rigorous logical operations.

For example, a car driver knows that if the car moves straight ahead then he does not need to do anything; if the car turns to the right then he needs to steer the car to the left; if the car turns to the right by too much then he needs to take a stronger action to steer the car to the left much more, and so on. Here, "much" and "more" etc. are fuzzy terms that cannot be described by classical mathematics but can be quantified by membership functions (see Fig. 2, where part (a) is an example of the description "to the left"). The collection of all such "if … then …" principles constitutes a *fuzzy logic rule base* for the problem under investigation. To this end, it is helpful to briefly summarize the experience of the driver in the following simplified rule base: Let X = [–180°, 180°], x be the position of the car, $\mu_{left}(\cdot)$ be the membership function for the moving car turning "to the left," $\mu_{right}(\cdot)$ the membership function for the car turning "to the right," and $\mu_0(\cdot)$ the membership function for the car "moving straight ahead." Here, simplified statements are used, for instance, "x is X_{left}" means "x belongs to X with a membership value $\mu_{left}(x)$" etc. Also, similar notation for the control action u of the driver is employed. Then, a simple typical rule base for this car-driving task is

$R^{(1)}$:	IF x is X_{left}	THEN u is U_{right}
$R^{(2)}$:	IF x is X_{right}	THEN u is U_{left}
$R^{(3)}$:	IF x is X_0	THEN u is U_0

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