

Fuzzy Approximation of DES State

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INTRODUCTION

State estimation of dynamic systems is a resort often used when only a subset of the state variables can be directly measured; observers are the entities computing the system state from the knowledge of its internal structure and its (partially) measured behaviour. The problem of discrete event systems (DES) estimation has been addressed in (Ramirez, 2003) and (Giua 2003); in these works the marking of a Petri net (PN) model of a partially observed event driven system is computed from the evolution of its inputs and outputs.

The state of a system can be also inferred using the knowledge on the duration of activities. However this task becomes complex when, besides the absence of sensors, the durations of the operations are uncertain; in this situation the observer obtains and revise a belief that approximates the current system state. Consequently this approach is useful for non critical applications of state monitoring and feedback in which an approximate computation is allowed.

The uncertainty of activities duration in DES can be handled using fuzzy PN (FPN) (Murata, 1996), (Cardoso, 1999), (Hennequin, 2001), (Pedrycz, 2003), (Ding, 2005); this PN extension has been applied to knowledge modelling (Chen, 1990), (Korriem, 2000), (Shen, 2003), planning (Cao, 1996), reasoning (Gao, 2003) and controller design (Andreu, 1997), (Leslaw, 2004).

In these works the proposed techniques include the computation of imprecise markings; however the class of models dealt does not include strongly connected PN for the modelling of cyclic behaviour. In this article we address the problem of state estimation of DES for calculating the fuzzy marking of a Fuzzy Timed Petri Net (FTPN); for this purpose a set of matrix expressions for the recursive computing the current fuzzy marking is developed. The article focuses on FTPN whose structure is a Marked Graph (called Fuzzy Timed Marked

Graph -FTMG) because it allows showing intuitively the problems of the marking estimation in exhibiting cyclic behaviour.

BACKGROUND

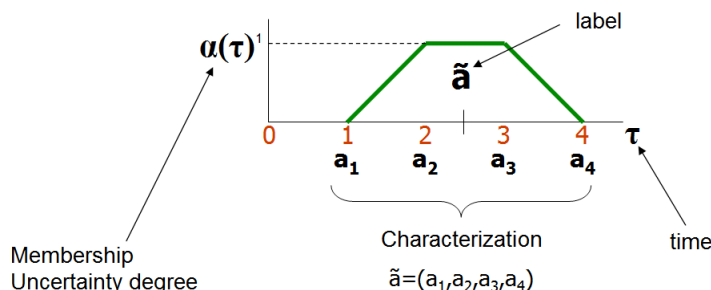
Possibility Theory

In theory of possibility, a fuzzy set \tilde{a} is used for delimiting ill-known values or for representing values characterized by symbolic expressions. The set is defined as $\tilde{a} = (a_1, a_2, a_3, a_4)$ such that $a_1, a_2, a_3, a_4 \in \mathbb{R}$, $a_1 \leq a_2$ and $a_3 \leq a_4$. The fuzzy set \tilde{a} delimits the run time as follows:

- The values $\tilde{\tau}_b, \tilde{\tau}_a$ in the ranges $(a_1, a_2), (a_3, a_4)$, respectively, indicate that the activity is possibly executed with $\alpha(\tau) \in (0, 1)$. When $\tau \in \tilde{\tau}_b$ the function $\alpha(\tau)$ grows towards 1, which means that the possibility of stopping increases. When $\tau \in \tilde{\tau}_a$, the membership function $\alpha(\tau)$ decreases towards 0, representing that there is a reduction of the possibility of stopping.
- The values $(0, a_1]$ mean that the activity is running.
- The values $[a_4, +\infty)$ mean that the activity is stopped.
- The values $\tilde{\tau}_a \in [a_2, a_3]$ | $a_2 \leq a_3$ represent full possibility that is $\alpha(\tau) = 1$, this represents that it is certain that the activity is stopped.
- The support of \tilde{a} is the range $\tau \in [a_1, a_4]$ where $\alpha_{\tilde{a}}(\tau) > 0$.

A fuzzy set \tilde{a} is referred indistinctly by the function $\alpha(\tau)$ or the characterization (a_1, a_2, a_3, a_4) . For simplicity, in this work the fuzzy possibility distribution of the time is described with trapezoidal or triangular forms. For example, Fig.1 shows the fuzzy set that

Figure 1. Fuzzy set



it is represents in natural language: “the activity will stop about 2.5”.

Fuzzy extension principle. The fuzzy extension principle plays a fundamental role because we can extend functions defined on crisp sets to functions on fuzzy sets. An important application of this principle is a mechanism to operate arithmetically with fuzzy numbers.

Definition. Let X_1, \dots, X_n be crisp sets and let f a function such $f: X_1 \times \dots \times X_n \rightarrow Y$. If $\tilde{a}_1, \dots, \tilde{a}_n$ are fuzzy sets on X_1, \dots, X_n , respectively, then $f(\tilde{a}_1, \dots, \tilde{a}_n)$ is the fuzzy set on Y such that:

$$f(\tilde{a}_1, \dots, \tilde{a}_n) = \cup_{(x_1, \dots, x_n) \in (X_1 \times \dots \times X_n)} \{ \alpha_{\tilde{a}_1}(x_1) \wedge \dots \wedge \alpha_{\tilde{a}_n}(x_n) / f(x_1, \dots, x_n) \}$$

If $\tilde{b} = f(\tilde{a}_1, \dots, \tilde{a}_n)$ then \tilde{b} is the fuzzy set on Y such that:

$$\tilde{b}(y) = \vee_{(x_1, \dots, x_n) \in (X_1 \times \dots \times X_n) : f(x_1, \dots, x_n) = y} [\alpha_{\tilde{a}_1}(x_1) \wedge \dots \wedge \alpha_{\tilde{a}_n}(x_n)]$$

The fuzzy set was characterized as:

$$\tilde{a} = \{ \alpha_{\tilde{a}}(x_1) / x_1, \dots, \alpha_{\tilde{a}}(x_n) / x_n \}$$

With the extension principle we can define a simplified fuzzy sets addition operation.

Definition. Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy sets. The fuzzy sets addition operation is: $\tilde{a} \oplus \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$ (Klir, 1995).

Definition The intersection and union of fuzzy sets are defined in terms of min and max operators.

$$(\tilde{a} \cap \tilde{b}) = \min(\tilde{a}, \tilde{b}) = \min(\alpha_{\tilde{a}}(\tau), \alpha_{\tilde{b}}(\tau)) \mid \tau \in \text{support_of_} \tilde{a} \wedge \tilde{b}$$

and

$$(\tilde{a} \cup \tilde{b}) = \max(\tilde{a}, \tilde{b}) = \max(\alpha_{\tilde{a}}(\tau), \alpha_{\tilde{b}}(\tau)) \mid \tau \in \text{support_of_} \tilde{a} \vee \tilde{b}$$

We used these operators, intersection and union, as a t-norm and a s-norm, respectively.

Definition The distribution of possibility before and after \tilde{a} are the fuzzy sets $\tilde{a}^b = (-\infty, a_2, a_3, a_4)$ and $\tilde{a}^a = (a_1, a_2, a_3, +\infty)$ respectively; they are defined in (Andreu, 1997) as a function $\alpha_{(-\infty, \tilde{a}_1]}(\tau) = \sup_{\tau' \geq \tau} \alpha(\tau')$ and $\alpha_{(\tilde{a}_n, +\infty]}(\tau) = \sup_{\tau' \leq \tau} \alpha(\tau')$, respectively.

Petri Nets Theory

Definition. An ordinary PN structure G is a bipartite digraph represented by the 4-tuple $G = (P, T, I, O)$ where $P = \{p_1, p_2, \dots, p_n\}$ and $T = \{t_1, t_2, \dots, t_m\}$ are finite sets of vertices called respectively places and transitions, $I(O): P \times T \rightarrow \{0, 1\}$ is a function representing the arcs going from places to transitions (transitions to places).

Pictorially, places are represented by circles, transitions are represented by rectangles, and arcs are depicted as arrows. The symbol $\bullet t_j (t_j \bullet)$ denotes the set

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