Complex Systems Modeling by Cellular Automata

Jiří Kroc
Section Computational Science, The University of Amsterdam, The Netherlands

Peter M. A. Sloot
Section Computational Science, The University of Amsterdam, The Netherlands

INTRODUCTION

In recent years, the notion of complex systems proved to be a very useful concept to define, describe, and study various natural phenomena observed in a vast number of scientific disciplines. Examples of scientific disciplines that highly benefit from this concept range from physics, mathematics, and computer science through biology and medicine as well as economy, to social sciences and psychology. Various techniques were developed to describe natural phenomena observed in these complex systems. Among these are artificial life, evolutionary computation, swarm intelligence, neural networks, parallel computing, cellular automata, and many others. In this text, we focus our attention to one of them, i.e. 'cellular automata'.

We present a truly discrete modelling universe, discrete in time, space, and state: Cellular Automata (CAs) (Sloot & Hoekstra, 2007, Kroc, 2007, Sloot, Chopard & Hoekstra, 2004). It is good to emphasize the importance of CAs in solving certain classes of problems, which are not tractable by other techniques. CAs, despite theirs simplicity, are able to describe and reproduce many complex phenomena that are closely related to processes such as self-organization and emergence, which are often observed within the above mentioned scientific disciplines.

BACKGROUND

We briefly explain the idea of complex systems and cellular automata and provide references to a number of essential publications in the field.

Complex Systems

The concept of complex systems (CSs) emerged simultaneously and often independently in various scientific disciplines (Fishwick, 2007, Bak, 1996, Resnick, 1997). This could be interpreted as an indication of their universality. Despite the diversity of those fields, there exist a number of common features within all complex systems. Typically a complex system consist of a vast number of simple and locally operating parts, which are mutually interacting and producing a global complex response. Self-organization (Bak, 1996) and emergence, often observed within complex systems, are driven by dissipation of energy and/or information.

Self-organization can be easily explained with ant-colony behavior studies where a vast number of identical processes, called ants, locally interact by physical contact or by using pheromone marked traces. There is no leader providing every ant with information or instructions what it should do. Despite the lack of such a leader or a hierarchy of leaders, ants are able to build complicated ant-colonies, feed their larvae, protect the colony, fight against other colonies, etc. All this is done automatically through a set of simple local interactions among the ants. It is well known that ants are responding on each stimuli by one out of 20 to 40 (depending on ant species) reactions, these are enough to produce the observed complexity.

Emergence is defined as the occurrence of new processes operating at a higher level of abstraction then is the level at which the local rules operate. Each level usually has its own local rules different from rules operating at other levels. An emergent, like an ant-colony, is a product of the process of emergence. There can be a whole hierarchy of emergents, e.g. as in the hu-

man body, that consists of chemicals and DNA, going through polypeptides, proteins, cellular infrastructures and cycles, further on to cells and tissues, organs, and bodies. We see that self-organization and emergence are often closely linked to one another.

Cellular Automata

Early development of CAs dates back to A. Turing, S. Ulam, and J. von Neumann. We can define CA's by four mutually interdependent parts: the lattice and its variables, the neighbourhood, and the local rules (Toffoli & Margolus, 1987, Toffoli, 1984, Vichniac, 1984, Ilachinski, 2001, Wolfram, 2002, Wolfram 1994, Sloot & Hoekstra, 2007, Kroc, 2007). This is briefly explained below.

Lattices and Networks

A lattice is created by a grid of elements, for historical reasons called cells, which can be composed in one, two, three, or higher dimensional space. The lattice is typically composed of uniform cells such as, for instance squares, hexagons or triangles in two dimensions.

CAs operating on networks and graphs represent a generalization of classical CAs, which are working on regular lattices. Networks can be random or regular. Networks can have various topologies, which are classified by the degree of regularity and randomness. A lattice of cells can be interpreted as a regular network of vertices interconnected by edges. When we leave this regularity and allow some random neighbours, more precisely, if a major part of a network is regular and a smaller fraction of it is random, then we enter the domain of small-world networks. The idea of small-world networks provides a unique tool, which allows us to capture many essential properties of naturally observed phenomena especially those linked to social networks and surprisingly to (metabolic and other) networks operating within living cells. Whereas small-world networks are a mixture of regular and random networks, pure random networks have a completely different scope of use. It is worth to mention the concept of scale-free networks, which have a connectivity that does not depend on scale anymore (Kroc, 2007, Sloot, Chopard & Hoekstra, 2004).

Variables

A CA contains an arbitrary number of discrete variables. The number and range of them are dictated by the phenomenon under study. The simplest CAs are built using only one Boolean variable in one dimension (1D), see e.g. (Wolfram, 2002). Some of such simple 1D CAs express even high complexity and are shown to be capable of the universal computation.

Neighbourhoods

The neighbourhood, which is used to evaluate a local rule, is defined by a set of neighbouring cells including the updated cell itself in the case of regular lattices, Figure 1. Neighbours with relative coordinates [i, j+1], [i-1,j], [i, j-1], [i+1, j] of the updated cell [i, j] and located on North, West, South, and East, respectively, define the so called the von Neumann neighbourhood with radius r =1. The Moore neighbourhood with radius r =1 contains the same cells as the von Neumann neighbourhood plus diagonal cells located at relative positions [i-1, j+1], [i-1, j-1], [i+1, j-1], [i+1, j+1], i.e. North-west, South-west, South-east, and North-east, respectively.

There are many other types of neighbourhoods possible; neighbourhoods can even be spatially or temporally non-uniform. One example is the Margolus neighbourhood, used in diffusion modelling.

The boundaries for each CA can be fixed, reflecting or periodic. Periodic boundary conditions represent infinite lattices. Periodic means that, e.g. in one dimension, the most right cell of a lattice is connected to the most left lattice cell. Fixed boundary cells are kept at predefined values. Reflecting boundary cells reflect values back to the bulk of the lattice.

Local Rules

A local rule defines the evolution of each CA. Usually; it is realized by taking all variables from all cells within the neighbourhood and by evaluation of a set of logical and/or arithmetical operations written in the form of an algorithm. The vector s of those variables is updated according to the following local rule in the case of the von Neumann neighbourhood

$$s[i,j] = f(s[i,j+1], s[i-1,j], s[i,j-1], s[i+1,j]),$$