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Chaotic Neural Networks

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INTRODUCTION

Artificial Neural Networks have proven, along the last four decades, to be an important tool for modelling of the functional structures of the nervous system, as well as for the modelling of non-linear and adaptive systems in general, both biological and non biological (Haykin, 1999). They also became a powerful biologically inspired general computing framework, particularly important for solving non-linear problems with reduced formalization and structure. At the same time, methods from the area of complex systems and non-linear dynamics have shown to be useful in the understanding of phenomena in brain activity and nervous system activity in general (Freeman, 1992; Kelso, 1995). Joining these two areas, the development of artificial neural networks employing rich dynamics is a growing subject in both arenas, theory and practice. In particular, model neurons with rich bifurcation and chaotic dynamics have been developed in recent decades, for the modelling of complex phenomena in biology as well as for the application in neuro-like computing. Some models that deserve attention in this context are those developed by Kazuyuki Aihara (1990), Nagumo and Sato (1972), Walter Freeman (1992), K. Kaneko (2001), and Nabil Farhat (1994), among others. The following topics develop the subject of Chaotic Neural Networks, presenting several of the important models of this class and briefly discussing associated tools of analysis and typical target applications.

BACKGROUND

Artificial Neural Networks (ANNs) is one of the important frameworks for biologically inspired computing. A central characteristic in this paradigm is the desire to bring to computing models some of the interesting properties of the nervous system such as adaptation, robustness, non-linearity, and the learning through examples.

When we focus on biology (real neural networks), we see that the signals generated in real neurons are used in different ways by the nervous system to code information, according to the context and the functionality (Freeman, 1992). Because of that, in ANNs we have distinct model neurons, such as models with graded activity based on frequency coding, models with binary outputs, and spiking models (or pulsed models), among others, each one giving emphasis to different aspects of neural coding and neural processing. Under this scenario, the role of neurodynamics is one of the target aspects in neural modelling and neuro-inspired computing; some model neurons include aspects of neurodynamics, which are mathematically represented through differential equations in continuous time, or difference equations in discrete time. As described in the following topic, dynamic phenomena happen at several levels in neural activity and neural assembly activity (in internal neural structures, in simple networks of interacting neurons, and in large populations of neurons). The model neurons particularly important for our discussion are those that emphasize the relationship between neurocomputing and non-linear dynamical systems with bifurcation and rich dynamic behaviour, including chaotic dynamics.

NEUROCOMPUTING AND THE ROLE OF RICH DYNAMICS

The presence of dynamics in neural functionality happens even at the more detailed cellular level: the well known Hodgkin and Huxley model for the generation and propagation of action potentials in the active membrane of real neurons is an example; time dependent processes related to synaptic activity and the post synaptic signals is another example. Dynamics also appears when we consider the oscillatory behaviour in real neurons under consistent stimulation. Additionally, when we consider neural assemblies, we also observe the emergence of important global dynamic behaviour for the production of complex functions.

As discussed ahead, non-linearity is an essential ingredient for complex functionality and for complex dynamics; there is a clear contrast between linear dynamic systems and non-linear dynamic systems, in what respect their potential for the production of rich and diverse behaviour.

Role of Non-Linear Dynamics in the Production of Rich Behaviour

In linear dynamical systems, both in continuous time and in discrete time, the autonomous dynamical behaviour is completely characterized through the system's natural modes, either the harmonic oscillatory modes, or the exponentially decaying modes (in the theory of linear dynamical systems, these are represented by frequencies and complex frequencies). The possible dynamic outcomes in linear systems are thus limited to the universe of linear combinations of these natural modes. These modes can have their properties of amplitudes and frequencies controlled through parameters of the system, but not their central properties such as the nature of the produced waveforms. Since the number of natural modes of linear systems is closely related to the number of state variables, we have that small networks (of linear dynamic elements) can produce only limited diversity of dynamical behaviour.

The scenario becomes completely different in non-linear systems. Non-linearity promotes rich dynamic behaviour, obtained by changing the stability and instability of different attractors. These changes give place to bifurcation phenomena (transitions between dynamic modalities with distinct characteristics) and therefore to diversity of dynamic behaviour. In non-linear systems, we can have a large diversity of dynamical behaviours, with the potential production of infinite number of distinct waveforms (or time series, for discrete time systems). This can happen for systems with very reduced number of state variables: just three in continuous time, or just one state variable in discrete time, are enough to allow bifurcation among different attractors and potential cascades of infinite bifurcations leading to chaos. In our context, this means obtaining rich attractor behaviour even from very simple neural networks (i.e., networks with a small number of neurons).

In summary, the operation of chaotic neural networks explores the concepts of attractors, repellers, limit cycles, and stability (see the topic Terms and Definitions for details on these concepts) of trajectories in the multidimensional state space of the neural network, and more specifically, the dense production of destabilization of cyclic trajectories with cascading to chaotic behaviour. This scenario allows for the blend of ordered behaviour and chaotic dynamics, and the presence of fractal structure and self-similarity in the rich landscape of dynamic attractors.

MODEL NEURONS WITH RICH DYNAMICS, BIFURCATION AND CHAOS

We can look at chaotic elements that compose neuro-like architectures from several different perspectives. They can be looked at as emergent units with rich dynamics that are produced by the interaction of classical model neurons, such as the sigmoidal model neurons based on frequency coding (Haykin, 1999), or the integrate and fire spiking model neurons (Farhat, 1994). They can also correspond to the modelling of dynamical behaviour of neural assemblies, approached as a unity (Freeman, 1992). Finally, they can be tools for approximate representation of aspects of complex dynamics in the nervous system, paying attention mainly to the richness of attractors and blend of ordered and erratic dynamics, and not exactly to the details of the biological dynamics (DelMoral, 2005; Kaneko, 2001). Ahead we describe briefly some of the relevant model neurons in the context of chaotic neural networks.

Aihara's Chaotic Model Neuron. One important work in the context of chaotic neural networks is the model neuron proposed by Kazuyuki Aihara and collaborators (1990). In it, we have self-feedback of the neuron's state variable, for representing the refractory period in real neurons. This makes possible rich bifurcation and cascading to chaos. His work extends previous models in which some elements of dynamics were already present. In particular, we have to mention the work by Caianiello (1961), in which the past inputs have impact on the value of the present state of the neuron, and the work by Nagumo and Sato (1972), which incorporates an exponential decay memory. Aihara's model included memory for the inputs of the model neuron as well as for its internal state. It also included continuous transfer functions, an essential ingredient for rich bifurcation, fractal structure and cascading to chaos. Equation 1 shows a simplified form