3D Printing: Basic Concepts
Mathematics and Technologies

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ABSTRACT

3D printing is about being able to print any object layer by layer. But if one questions this proposition, can one find any three-dimensional objects that can’t be printed layer by layer? To banish any disbeliefs the authors walked together through the mathematics that prove 3d printing is feasible for any real life object. 3d printers create three-dimensional objects by building them up layer by layer. The current generation of 3d printers typically requires input from a CAD program in the form of an STL file, which defines a shape by a list of triangle vertices. The vast majority of 3d printers use two techniques, FDM (Fused Deposition Modelling) and PBP (Powder Binder Printing). One advanced form of 3d printing that has been an area of increasing scientific interest the recent years is bioprinting. Cell printers utilizing techniques similar to FDM were developed for bioprinting. These printers give us the ability to place cells in positions that mimic their respective positions in organs. Finally, through a series of case studies the authors show that 3d printers have made a massive breakthrough in medicine lately.

Keywords: 3D Printing, Bionic Ear, Bioprinting, Fubini Theorem, Fused Deposition Modelling (FDM), Human Heart, Organ Printing, Powder Binder Printing (PBP), Skull Lesions

INTRODUCTION

3d printing is a rapidly developing technology. Such an industrial revolution could have many applications in the fields of engineering, medicine and much more. These include creation of mass-customized products, prototypes, replacement parts and even medical and dental implants. The speed and ease of designing and modifying products has made them the number one prototyping technique. The aim of this article is to evaluate this matter from another point of view by focusing on

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their contribution to the field of medicine. We will first examine the mathematical principles behind 3d printing. Next we will give a short description of the most famous 3d printing methods highlighting one special form or it, called bioprinting. Finally we will focus on a series of case studies where 3d printers where used to guide surgeons, create enhanced human parts and mend human skull defects.

3D PRINTING MADE REAL: FUBINI THEOREM

Before we turn our focus on printing a three-dimensional model we have to consider what mathematical statements and theorems allows us to materialize this idea. Practically 3d printing is about being able to print any object layer by layer. But if we question this belief, can we find any three-dimensional objects that can’t be printed layer by layer?

So next we will be walking through the theorem that proves 3d printers can duplicate everything (any real life physical object at least). Fubini’s theorem, named after the Italian mathematician Guido Fubini, states that an object of \( n \) dimensions can be represented as a spectrum of layers of shapes of \( n-1 \) dimensional layers. This means that a 3 dimensional shape (any shape in the real world) can be portrayed as layers of 2 dimensional shapes (3dfuture, 2012). In 3d printing technology this means that we are able to express any 3d object as layers of 2d planes. Below we provide the theorem but not its proof since it doesn’t serve the purpose of this article. Readers are referred in analysis 3 as described in (Tsirelson, 2011; Zakeri, 2007).

**Theorem Statement**

Suppose \( A \) and \( B \) are complete measure spaces. Supposes \( f(x,y) \) is \( A \times B \) measurable. If

\[
\int_{A \times B} |f(x,y)| \, d(x,y) < \infty,
\]

where the integrals is taken with respect to a product measure on the space over \( A \times B \), then

\[
\int_{A} \left( \int_{B} f(x,y) \, dy \right) \, dx = \int_{B} \left( \int_{A} f(x,y) \, dx \right) \, dy = \int_{A \times B} f(x,y) \, d(x,y),
\]

The first two integrals being iterated integrals with respect to two measures, respectively and the third being an integral with respect to a product of these two measures.

If the above integral of the absolute value is not finite, then the two iterated integrals may actually have different values. See below for an illustration of this possibility.

**Corollary**

If \( f(x,y)=g(x)h(y) \) for some functions \( g \) and \( h \), then

\[
\int_{A} \int_{B} g(x)h(y) \, dy \, dx = \int_{A \times B} f(x,y) \, d(x,y),
\]

The integral on the right side being with respect to a product measure.

**Alternate Theorem Statement**

Another version of Fubini’s theorem states that if \( A \) and \( B \) are \( \sigma \)-finite measure spaces, not necessarily complete, and if either

\[
\int_{A} \left( \int_{B} |f(x,y)| \, dy \right) \, dx < \infty \quad \text{or} \quad \int_{B} \left( \int_{A} |f(x,y)| \, dx \right) \, dy < \infty,
\]

then

\[
\int_{A \times B} |f(x,y)| \, d(x,y) < \infty,
\]

and
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