

Formal Rules for Fuzzy Causal Analyses and Fuzzy Inferences

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ABSTRACT

Causal inference is one of the central capabilities of the natural intelligence that plays a crucial role in thinking, perception, and problem solving. Fuzzy inferences are an extended form of formal inferences that provide a denotational mathematical means for rigorously dealing with degrees of matters, uncertainties, and vague semantics of linguistic variables, as well as for rational reasoning the semantics of fuzzy causalities. This paper presents a set of formal rules for causal analyses and fuzzy inferences such as those of deductive, inductive, abductive, and analogical inferences. Rules and methodologies for each of the fuzzy inferences are formally modeled and illustrated with real-world examples and cases of applications. The formalization of fuzzy inference methodologies enables machines to mimic complex human reasoning mechanisms in cognitive informatics, cognitive computing, soft computing, abstract intelligence, and computational intelligence.

Keywords: Abduction, Causal Analysis, Causality Network, Cognitive Computing, Cognitive Informatics, Computational Intelligence, Deduction, Denotational Mathematics, Formal Inference Rules, Fuzzy Inference, Fuzzy Logic, Fuzzy Rules, Induction, Soft Computing

1. INTRODUCTION

A *causation* is a relationship between a sole or multiple causes and a single or multiple effects. The *cause* in a causation is a premise state such as an event, phenomenon, action, behavior, or existence; while the *effect* is a consequent or conclusive state such as an event, phenomenon, action, behavior, or existence.

An *inference* is a cognitive process that deduces a conclusion, particularly a causation, based on evidences and reasoning. Causal inference is one of the central capabilities of human brains that plays a crucial role in thinking, perception, and problem solving (BISC,

2008; Sternberg, 1998; Payne & Wenger, 1998; Zadeh, 1975, 1998, 1999; Smith, 2001; Wang, 2002a, 2007b, 2008b, 2009a; Wang et al., 2006, 2009). The framework of causal inferences can be classified into four categories known as the *intuitive, empirical, heuristic, and rational* causalities (Wang, 2011a, 2011c, 2012c). Therefore, a causal inference can be conducted based on empirical observations, formal inferences, and/or statistical regulations (Bender, 1996; Wilson and Keil, 2001; Wang, 2007a, 2007b, 2008a, 2011a, 2011c, 2012c).

Formal logic inferences may be classified as deductive, inductive, abductive, and analogical inferences (Schoning, 1989; Sperschneider &

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Antoniou, 1991; Rose, 1995; Hurley, 1997; van Heijenoort, 1997; Tomassi, 1999; Smith, 2001; Wilson & Keil, 2001; Wang, 2007b; Wang et al., 2006, 2009, 2011xx). Although logic inferences can be carried out on the basis of abstraction and symbolic reasoning with crisp sets and Boolean logic, more human inference mechanisms and rules such as those of intuitive, empirical, heuristic, and perceptive inferences are fuzzy and uncertain, which are yet to be studied by fuzzy inferences on the basis of fuzzy sets and fuzzy logic (Zadeh, 1965, 1971, 1975, 1983, 2004, 2006, 2008; Wang, 2008a) in the context of denotational mathematics (Wang, 2002b, 2007d, 2008a, 2008c, 2009c, 2011a, 2011c, 2012a, 2012b, 2012c, 2012h; Wang & Chiew, 2011) and cognitive computing (Wang, 2002a, 2003, 2007c, 2007e, 2008d, 2008e, 2009b, 2009d, 2009e, 2010a, 2010b, 2011b, 2011d, 2012d, 2012e, 2012f, 2012g, 2012i, 2012j, 2012k, 2012l, 2013a, 2013b, 2013c; Wang & Wang, 2006; Wang & Fariello, 2012; Wang et al., 2006, 2009, 2010a, 2010b, 2012).

Typical fuzzy inferences, rules, and applications will be formally modeled in this paper encompassing the deductive, inductive, abductive, and analogical inferences. In the remainder of this paper, the basic rules for causality analyses are described in Section 2. Then, each of the above fuzzy inference processes is rigorously modeled and elaborated in Section 3 through Section 6. Case studies and examples are provided for illustrating applications of the mathematical structures of fuzzy inference rules and methodologies.

2. RULES OF CAUSALITIES AND INFERENCE

Causalities are a universal phenomenon because any event, phenomenon, action, or behavior has a cause. Any sequence of events, phenomena, actions, or behaviors may be identified as a series of causal relations. This section explores the taxonomy of causalities and their generic properties (Wang, 2011a, 2011c, 2012c). The concept of causality networks and a framework

of causal inferences are described, before they are extended to fuzzy causal inferences.

2.1. Taxonomy and Rules of Causality

As a preparation for the following subsections, the taxonomy and rules of logical causality are described below.

Definition 1: A *binary causality* κ_b is a binary relation that links a pair of events or states as the cause P and effect Q , i.e.:

$$\kappa_b \mathbf{BL} \triangleq (P\mathbf{BL} \vdash Q\mathbf{BL})\mathbf{BL} \tag{1}$$

where \vdash denotes yield and the causality and propositions are in type Boolean as denoted by the type suffix **BL**. Hence, $\kappa_b \mathbf{BL} = \mathbf{T}$ called a *valid causality*, otherwise it is a *fallacy*, i.e., $\kappa_b \mathbf{BL} = \mathbf{F}$.

Equation 1 can also be denoted in a vertical structure as follows:

$$\kappa_b \mathbf{BL} \triangleq \left| \frac{Premises\mathbf{BL}}{Conclusion\mathbf{BL}} \right| = \left| \frac{P\mathbf{BL}}{Q\mathbf{BL}} \right| \tag{2}$$

The binary causality as given in Equations 1 and 2 can be extended to the n -nary, chain, reflective, and loop causalities as follows.

Definition 2: An *n-nary causality* κ_n is a composite form of binary causality where multiple causes $P_1\mathbf{BL}, P_2\mathbf{BL}, \dots, P_n\mathbf{BL}$, mutually result in an effect, i.e.:

$$\begin{aligned} \kappa_n \mathbf{BL} &\triangleq ((P_1\mathbf{BL} \wedge P_2\mathbf{BL} \wedge \dots \wedge P_n\mathbf{BL}) \vdash Q\mathbf{BL})\mathbf{BL} \\ &= \left(\bigwedge_{i=1}^n P_i\mathbf{BL} \vdash Q\mathbf{BL} \right)\mathbf{BL} \end{aligned} \tag{3}$$

Definition 3: A *chain causality* κ_c is a series of causalities where each state Q_i , as an effect of a preceding cause, is the cause of a succeeding effect, i.e.:

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