Inference Algebra (IA):
A Denotational Mathematics for Cognitive Computing and Machine Reasoning (II)

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ABSTRACT

Inference as the basic mechanism of thought is abilities gifted to human beings, which is a cognitive process that creates rational causations between a pair of cause and effect based on empirical arguments, formal reasoning, and/or statistical norms. It's recognized that a coherent theory and mathematical means are needed for dealing with formal causal inferences. Presented is a novel denotational mathematical means for formal inferences known as Inference Algebra (IA) and structured as a set of algebraic operators on a set of formal causations. The taxonomy and framework of formal causal inferences of IA are explored in three categories: a) Logical inferences; b) Analytic inferences; and c) Hybrid inferences. IA introduces the calculus of discrete causal differential and formal models of causations. IA enables artificial intelligence and computational intelligent systems to mimic human inference abilities by cognitive computing. A wide range of applications of IA are identified and demonstrated in cognitive informatics and computational intelligence towards novel theories and technologies for machine-enabled inferences and reasoning. This work is presented in two parts. The inference operators of IA as well as their extensions and applications will be presented in this paper; while the structure of formal inference, the framework of IA, and the mathematical models of formal causations has been published in the first part of the paper in IJCINI 5(4).

Keywords: Abstract Intelligence, Applications, Causal Differential, Cognitive Computers, Cognitive Computing, Cognitive Informatics, Computational Intelligence, Denotational Mathematics, Formal Causations, Inference Algebra, Inference Engine

1. INTRODUCTION

Inference is a reasoning process that derives a causal conclusion from given premises. Formal inferences are usually symbolic and mathematical logic based, in which a causation is proven true by empirical observations, logical truths, mathematical equivalence, and/or statistical norms. Conventional logical inferences may be classified into the categories of deductive, inductive, abductive, and analogical inferences (Zadeh, 1965, 1975, 1999, 2004, 2008; Schoning, 1989; Sperschneider & Antoniou, 1991; Hurley, 1997; Tomassi, 1999; Wilson & Clark, 1988; Wang, 2007b, 2008a, 2011a; Wang, Wang, Patel, & Patel, 2006), as well as qualification and quantification (Zadeh, 1999, 2004; Wang, 2007b, 2009c). Studies on mechanisms and laws of inferences...
can be traced back to the very beginning of human civilization, which formed part of the foundations of various disciplines such as philosophy, logic, mathematics, cognitive science, artificial intelligence, computational intelligence, abstract intelligence, knowledge science, computational linguistics, and psychology (Zadeh, 1965, 1975, 2008; Mellor, 1995; Ross, 1995; Bender, 1996; Leahey, 1997; Wang, 2007c).

Although there are various inference schemes and methods developed in a wide range of disciplines and applications, the framework of formal inferences can be described in five categories known as the relational, rule-based, logical, fuzzy logical, and causal inferences. With a higher expressive power, causal inferences are a set of advanced inference methodologies building upon the other fundamental layers, which is one of the central capabilities of human brains which plays a crucial role in thinking, perception, reasoning, and problem solving (Zadeh, 1975; BISC, 2010; Wang, 2009c, 2011a; Wang, Zadeh, & Yao, 2009). The coherent framework of formal inferences reveals how human reasoning may be formalized and how machines may rigorously mimic the human inference mechanisms. The central focus of formal inferences is to reveal causations implied in a thread of thought beyond the semantics of a natural language expression.

**Definition 1.** Let \( \mathcal{S} \) be a finite nonempty set of states or facts that consists of both sets of causes \( \mathcal{C} \) and effects \( \mathcal{E} \), and \( \mathcal{R} \) a finite nonempty set of causations. The universal inference environment \( \mathcal{U} \), which forms the discourse of causality, is denoted as a triple, i.e.:

\[
\mathcal{U} \triangleq (\mathcal{C}, \mathcal{E}, \mathcal{R})
\]

where \( \mathcal{R} \) is a Cartesian product of causes and effects,

\[
\mathcal{R} = \mathcal{C} \times \mathcal{E}, \quad \mathcal{C} \subseteq \mathcal{S} \subseteq \mathcal{U}, \quad \mathcal{E} \subseteq \mathcal{S} \subseteq \mathcal{U}
\]

On the basis of the causal discourse \( \mathcal{U} \), a causation is a relation of a logical consequence between a sole or multiple causes and a single or multiple effects. A causation is usually a pair such as \((\text{Cause}, \text{Effect})\). The causal relations may be 1-1, 1-n, n-1, and n-m, where \( n \) and \( m \) are integers greater than 1 that represent multiple relations. The cause \( C \) in a causation on \( \mathcal{U} \) is a premise state such as an event, phenomenon, action, behavior, or existence. Related to a cause, a reason is a premise of an argument in support of a belief or causation. However, the effect \( E \) in a causation on \( \mathcal{U} \) is a consequent or conclusive state such as an event, phenomenon, action, behavior, or existence.

**Definition 2.** An abstract causation \( \zeta \), \( \zeta \in \mathcal{B} \mathcal{R} \subseteq \mathcal{U} \), in the discourse of causality \( \mathcal{U} \), is a relation that maps a set of causes \( C, C \subseteq \mathcal{C} \subseteq \mathcal{U} \) onto a set of effects \( E, E \subseteq \mathcal{E} \subseteq \mathcal{U} \), i.e.:

\[
\zeta \triangleq f_{\zeta} : C \rightarrow E, \quad C \subseteq \mathcal{C} \subseteq \mathcal{U}, E \subseteq \mathcal{E} \subseteq \mathcal{U}
\]

Based on the abstract causations \( \mathcal{Z} \) in the discourse of causality \( \mathcal{U} \), a formal inference can be explained as a cognitive process that deduces a conclusion, particularly a causation, based on evidences and reasoning.
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