

# Chapter 20

## Some Remarks on the Concept of Approximations from the View of Knowledge Engineering

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### ABSTRACT

*The concepts of approximations in granular computing (GrC) vs. rough set theory (RS) are examined. Examples are constructed to contrast their differences in the Global GrC Model (2<sup>nd</sup> GrC Model), which, in pre-GrC term, is called partial coverings. Mathematically speaking, RS-approximations are “sub-base” based, while GrC-approximations are “base” based, where “sub-base” and “base” are two concepts in topological spaces. From the view of knowledge engineering, its meaning in RS-approximations is rather obscure, while in GrC, it is the concept of knowledge approximations.*

### INTRODUCTION

Approximation is a serious concept in rough set theory (RS); it defines the rough sets. While in granular computing (GrC), it can be considered from three semantic views: Knowledge Engineering (KE), Uncertainty mathematics, and how-to-compute/solve-it [5]. Each view will have its own theory. In this paper, we will focus on KE view.

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This paper is a continuous effort that was initiated in Lin (2006b) and Barot and Lin (2008).

### RS-APPROXIMATIONS IN (INFINITE) UNIVERSE

The approximation theory of RS is well known. For preciseness, we will recall the notion here. Let  $U$  be a classical set, called the universe. Let  $\beta$  be a partition, namely, a family of sub-

sets, called equivalence classes, that are mutually disjoint and their union is the whole universe  $U$ . Then the pair  $(U, \beta)$  is called approximation space in RS. Pawlak introduced following two definitions. Observe that Pawlak focus on finite universe. However we allow  $U$  to be infinite.

Let  $X$  be an arbitrary subset of the universe  $U$ .

**Definition (RS) 1** Let  $E$  be an arbitrary equivalence class of  $R$ .

Upper approximation:

$$U[X] = \cup \{E \mid E \cap X \neq \emptyset\}$$

Lower approximation:

$$L[X] = \cup \{E \mid E \subseteq X\}$$

This definition is the formal form of the intuitive upper and lower approximations

**Definition (RS) 2** Let  $p$  be an arbitrary element of  $U$ .

Closure

$C[X] = \{p \mid \forall E, \text{ if } p \in E, \text{ then } E \cap X \neq \emptyset\}$ ;  
note that  $C[X]$  is a closed set in the sense of topological spaces.

Interior

$$I[X] = \{p \mid \exists E \text{ such that } p \in E \text{ \& } E \subseteq X\}.$$

In RS community, the previous definitions are directed generalized to Covering  $Cov$  by interpreting  $E$  as member of  $Cov$ .

## COUNTER INTUITIVE PHENOMENA

In this section, we present some Counter Intuitive phenomena of approximations. The first example was generated to answer some questions raised in

a conversation with Tian Yang, Guangming Lang, Jing Hao from Hunan University.

**Example 1.** Let the universe  $U$  be the real line.

Let us consider the collection  $COV$  of all open half lines, namely, the sets of the following form  $\{u \mid u < a\}$  and  $\{u \mid a < u\}$  for  $a \in U$ . These half lines form a sub-base of the usual topology in real line; Here the “usual topology” is a technical term referring to the topology of commonly known closure of the whole set.

In RS community, the approximations are defined by Definitions RSD1 or RSD2 given below, based on such definitions, one can readily see that the upper approximation of any finite interval, in fact any bounded sub-set, is the whole real line and its lower approximations is empty set.

- The RS-approximation space of a covering is too coarse.

This is an example of infinite universe; we will transform it into a finite universe.

**Example 2.** By considering any finite interval of integers, we have a similar example for a finite universe. Let  $U$  be set of the integers, say  $[1, 1000]$ . Let  $COV \cap U$  be the collection of all open half lines restricted to  $U$ . Then any open sub-interval, say  $(a, b)$ , has the itself as upper approximation and empty set as lower approximation.

The approximation spaces of RS and some GrC models are actually NS-spaces, where NS is neighborhood systems (Barot & Lin, 2008; Lin, 1989a; Lin, 1989b; Lin, 2009a; Lin, 2009b). For convenience of readers, we recall the definition of NS-space here

**Definition (NS) 3:** The pair  $(U, \beta)$  is called a NS-space, if  $\beta$  associates to each point of  $U$  a family of subsets of  $U$ , formally

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