Chapter 7
On the System Algebra Foundations for Granular Computing

Yingxu Wang
University of Calgary, Canada

Lotfi A. Zadeh
University of California, Berkeley, USA

Yiyu Yao
University of Regina, Canada

ABSTRACT

Granular computing studies a novel approach to computing system modeling and information processing. Although a rich set of work has advanced the understanding of granular computing in dealing with the "to be" and "to have" problems of systems, the "to do" aspect of system modeling and behavioral implementation has been relatively overlooked. On the basis of a recent development in denotational mathematics known as system algebra, this paper presents a system metaphor of granules and explores the theoretical and mathematical foundations of granular computing. An abstract system model of granules is proposed in this paper. Rigorous manipulations of granular systems in computing are modeled by system algebra. The properties of granular systems are analyzed, which helps to explain the magnitudes and complexities of granular systems. Formal representation of granular systems for computing is demonstrated by real-world case studies, where concrete granules and their algebraic operations are explained.

DOI: 10.4018/978-1-4666-0261-8.ch007

INTRODUCTION

The term *granule* is originated from Latin *granum*, i.e., grain, to denote a small compact particle in physics and in the natural world. The *taxonomy of granules* in computing can be classified into the data granule, information granule, concept granule, computing granule, cognitive granule, and system granule (Zedeh, 1979, 2003; Lin, 1998; Skowron and Stepaniuk, 2001; Yao, 2001, 2004a; Wang, 2007a, 2008c). The study of granular computing as an emerging filed appeared in 1997 (Zadeh, 1997, 1998; Lin, 1998). Granular computing may be viewed as an umbrella term covering theories, strategies, methodologies, techniques, tools, and systems that explore multilevel granularity in information processing, knowledge manipulation, and problem solving (Yao, 2001, 2004a, 2004b, 2005).

The concept of granules in data and information modeling and its fuzzy set treatment can be traced back to the work of L.A. Zedeh in 1979 as given below (Zadeh, 1979, 2003).

Definition 1. *The data granule g is a set with the elements x as a member of a fuzzy set $\tilde{G}$ to the degree of λ, 0 ≤ λ ≤ 1, i.e.:*

$$g \triangleq \{x \mid x \in_\lambda \tilde{G} \subseteq U\} \tag{1}$$

where U is the universal discourse.

Many studies investigated into granular computing based on rough sets (Lin, Yao, and Zadeh, 2002). Pawlak (1998) studied *knowledge granularity* using rough sets. Skowron and Stepaniuk (2001) proposed a rough set treatment of *information granules*. Polkowski and Skowron (1998) introduced the *granular calculus*. Lin (1998) studied *relational granules*. Pedrycz (2001) as well as Bargiela and Pedrycz (2002) suggested that granular computing may adopt a pyramid model toward various information granulations. Yao developed a trarachic perspective on granular computing with the facets of philosophy, methodology, and computational implementation (Yao, 2001, 2004a, 2005), which explains the structures of granular computing by multiple levels and views. These studies have advanced the theories of granular computing in dealing with the aspects of system "*to be*" and "*to have*" problems, particularly system architectures and high-level system conceptual designs in computing, software engineering, system engineering, and cognitive informatics. Wang initiated a set of *denotational mathematics* (Wang, 2002b, 2007a, 2007c, 2007d, 2008a) known as *concept algebra* (Wang, 2008b), *system algebra* (Wang, 2008c), and *Real-Time Process Algebra* (RTPA) (Wang, 2002a, 2003b, 2007a, 2008d), which were recognized as an expressive mathematical means for modeling and manipulating all types of granules in granular computing such as the *computing, cognitive, concept, information, data granules*, and *knowledge* granules.

This article presents a new perspective on the system metaphor of granules and granular computing, which extends the conventional set metaphors (Zadeh, 1979; Klir, 1992; Wang, 2007a). The following discusses the relationships between granules/systems and granular computing/system algebra. It will demonstrate that systems may be treated rigorously as a new mathematical structure beyond conventional mathematical entities. Based on this view, the concept of granules and granular computing are discussed below.

Definition 2. *A computing granule, shortly a granule, is a basic mathematical structure that possesses a stable topology and at least a unit of computational capability or behavior.*

It is noteworthy that, comparing Definitions 1 and 2, the computing granule is not a set, but an abstract system (Wang, 2008c) with both a given structure and a set of certain behaviors. The

Related Content