Chapter 3
PDE-Based Image Processing:
Image Restoration

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ABSTRACT

This chapter describes the basic concepts of partial differential equations (PDEs) based image modelling and their applications to image restoration. The general basic concepts of partial differential equation (PDE)-based image modelling and processing techniques are discussed for image restoration problems. These techniques can also be used in the design and development of efficient tools for various image processing and vision related tasks such as restoration, enhancement, segmentation, registration, inpainting, shape from shading, 3D reconstruction of objects from multiple views, and many more. As a case study, the topic in consideration is oriented towards image restoration using PDEs formalism since image restoration is considered to be an important pre-processing task for 3D surface geometry, reconstruction, and many other applications. An image may be subjected to various types of noises during its acquisition leading to degraded quality of the image, and hence, the noise must be reduced. The noise may be additive or multiplicative in nature. Here, the PDE-based models for removal of both types of noises are discussed. As examples, some PDE-based schemes have been implemented and their comparative study with other existing techniques has also been presented.

INTRODUCTION

Image processing is a rapidly growing field which can be defined as the manipulation of an image for the purpose of either extracting information from the image or producing an alternative representation of the image. The scientific structure of any image can be supposed to be based on an intrinsic principles of mathematics i.e. from image analysis to image processing. Image analysis includes modeling and analysis of original image itself, i.e. from image space analysis to different methods to represent image. The various tools of image analysis include spectral analysis, wavelets, statistics, level-sets and partial differential equations (PDEs). On the other hand, image processing is to modify the original image to improve the quality or extracting information from the given

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image, for example, image restoration, compression, segmentation, shape and texture analysis. There are two dual fields that are directly connected to the image processing in contemporary Computer science. These are Computer vision which is related to the construction of the 3D world from the observed 2D images; and another one is Computer graphics which pursues the opposite direction in designing suitable 2D scene images to simulate our 3D world. Image processing can be considered as the crucial middle way connecting the vision and graphics fields. Image processing can be considered as an input-output system shown in Figure 1.

I_0 is the input data which represents an observed or measured single image or image sequences; τ denotes a typical image processor e.g. restoration, enhancement, segmentation, compression, interpolation, feature extraction, inpainting etc.; and I denotes output which is also an image or an image sequence $(I_1, I_2, I_3, \ldots..)$ that contains all the targeted image features. The problem very often encountered in this area is to design an efficient and cost effective and accurate image processor. Typical design tasks include: de-noising, deblurring, edge detection, intensity enhancement, inpainting, interpolation, compression and decompression etc. In addition to these relatively low-level tasks, there are mid- and high-level tasks like disocclusion, shape from shading, motion analysis, image segmentation, and pattern identification and recognition. In image representation (Jain, 2006), one is concerned with characterization of the quantity that each picture element or pixel represents. An image could represent luminance of objects in a scene such as in digital photography, the absorption characteristics of the body tissue as in X-Ray imaging, the radar cross section of a target in radar imaging, the temperature profile of a region in infrared imaging or the gravitational field in an area in geophysical imaging. In general, any 2D function that contains information can be considered as an image. Image models give a logical or quantitative description of the properties of this function. There are three crucial ingredients of image processing which include modeling, analysis, and efficient implementation of processing tools. The design of a successful image processing technique relies on having a successful model for images. A fundamental issue faced in the design of image analysis techniques is the identification and characterization of the image space. The various approaches to *image modeling and analysis* include statistical representations or random field modeling; spectral and wavelet representations; and scale-space representations or regularity spaces.

The statistical approaches treat images as samples from random fields, which are often modeled by Markov or Gibbs fields or via statistical learning from an image database. The statistical properties of the fields are often established through the filtering technique and learning theory. Statistical models describe an image as a member of an ensemble, often characterized by its mean and covariance functions. This permits development of algorithms that are useful for an entire class or an ensemble of images rather than for a single image. Often the ensemble is assumed to be stationary so that the mean and covariance functions can easily be estimated. Random field modeling is the most appropriate approach for describing natural images with rich texture patterns

Figure 1. An image processing system