Chapter XXI
On Complex Artificial Higher Order Neural Networks:
Dealing with Stochasticity, Jumps and Delays

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ABSTRACT

This chapter deals with the analysis problem of the global exponential stability for a general class of stochastic artificial higher order neural networks with multiple mixed time delays and Markovian jumping parameters. The mixed time delays under consideration comprise both the discrete time-varying delays and the distributed time-delays. The main purpose of this chapter is to establish easily verifiable conditions under which the delayed high-order stochastic jumping neural network is exponentially stable in the mean square in the presence of both the mixed time delays and Markovian switching. By employing a new Lyapunov-Krasovskii functional and conducting stochastic analysis, a linear matrix inequality (LMI) approach is developed to derive the criteria ensuring the exponential stability. Furthermore, the criteria are dependent on both the discrete time-delay and distributed time-delay, hence less conservative. The proposed criteria can be readily checked by using some standard numerical packages such as the Matlab LMI Toolbox. A simple example is provided to demonstrate the effectiveness and applicability of the proposed testing criteria.

INTRODUCTION

Artificial neural networks are known to have successful applications in pattern recognition, pattern matching and mathematical function approximation. Comparing to the traditional first-order neural networks, artificial higher order neural networks (HONNs) allow high-order interactions between neurons, and therefore have stronger approximation property, faster convergence rate, greater storage capacity, and higher fault tolerance, see Artyomov & Yadid-Pecht (2005), Dembo et al. (1991), Karayiannis & Venetsanopoulos (1995), Lu et al (2006), and Psaltis et al. (1988). As pointed out in Giles & Maxwell (1987), HONNs have been shown to have impressive learning capability because the order or structure of a HONN can be tailored to the order or structure of the problem and also the knowledge can be encoded in HONNs. Due to the fact that time delays exist naturally in neural processing and signal transmission (Arik, 2005; Cao & Chen, 2004; Cao et al., 2005; Zhao, 2004a; Zhao, 2004b), the stability analysis problems for HONNs with discrete and/or distributed delays have drawn particular research attention, see e.g. Cao et al. (2004), Ren and Cao (2006), Wang et al. (2007) and Xu et al. (2005) for some recent results.

In real nervous systems, the synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes (Kappen, 2001). Indeed, such a phenomenon always appears in the electrical circuit design when implementing the neural networks. Also, the nervous systems are often subjected to external perturbations which are of a random nature. Stochastic neural networks have been extensively applied in many areas, such as pattern classification (Kappen, 2001) and time series prediction (Lai and Wong, 2001). In Kappen (2001), the application of stochastic neural networks based on Boltzmann machine learning has been demonstrated on a digit recognition problem, where the data consists of 11000 examples of handwriting digits (0-9) complied by the U.S. Postal Service Office of Advanced Technology, and the examples are pre-processed to produce 8 binary images. The main idea in Kappen (2001) is to model each of the digits using a separate Boltzmann Machine with a flat stochastic distribution, which gives rise to a special kind of stochastic neural networks with stochastically binary neurons. It has been shown in Kappen (2001) that the classification error rate for the test data set of handwriting digits using the stochastic neural networks is much lower than that using the traditional neural networks (e.g. nearest neighbour, back-propagation, wake-sleep, sigmoid belief). Furthermore, in Lai and Wong (2001), the stochastic neural network has been used to approximate complex nonlinear time series with much lower computational complexity than those for conventional neural networks, and the stochastic neural networks have been shown in Lai and Wong (2001) to have the universal approximation property of neural networks, and successfully improve post-sample forecasts over conventional neural networks and other nonlinear and nonparametric models. In addition, it has recently been revealed in Blythe et al. (2001) that a neural network could be stabilized or destabilized by certain stochastic inputs. Therefore, it is of practical significance to study the stability for delayed stochastic neural networks, and some preliminary results have been published, for example, in Huang et al. (2005), Wan and Sun (2005), Wang et al. (2006a), Wang et al. (2006b), Wang et al. (2006c) and Zhang et al. (2007). Note that, in Wang et al. (2006a) and Wang et al. (2006b), both the discrete and distributed time delays have been taken into account in the stochastic neural network models.

On the other hand, neural networks often exhibit a special characteristic of *network mode switching*. In other words, a neural network sometimes has finite modes that switch from one to another at different times (Casey, 1996; Huang et