An Efficient and Simple Algorithm for Matrix Inversion

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ABSTRACT

In this article, a new algorithm is proposed for finding inverse and determinant of a given matrix in one instance. The algorithm is straightforward in understanding and manual calculations. Computer implementation of the algorithm is extremely simple and is quite efficient in time and memory utilization. The algorithm is supported by an example. The number of multiplication/division performed by the algorithm is exactly \( n^3 \), however, its efficiency lies in the simplicity of coding and minimal utilization of memory. Simple applicability and reduced execution time of the method is validated form the numerical experiments performed on test problems. The algorithm is applicable in the cases of pseudo inverses for non-square matrices and solution of system of linear equations with minor modification.

Keywords: Algorithm, Linear Algebra, Matrix Inversion, Pseudo Inverse

INTRODUCTION

The problem of finding inverse is one of the important problems in applied sciences and engineering. There are several methods available for finding inverses and for solving system of linear equations like iterative methods, Gauss elimination procedure and decomposition methods (Burden, 2001; Fill, 1997; Najafi, 2006; Rao, 1971). Recently many researchers worked on the area of matrix inversion (Chang, 2006; Mikkawy, 2006; Vajargah, 2007). If inverse of coefficient matrix \( A \) in a given linear system is known then the solution can be found by \( X = A^{-1}b \).

In fact, the inversion of matrix is more generic requirement than the solution of linear system of equations in numerical linear algebra. Many algorithms exist to obtain numerical inversion for the given nonsingular matrix. A survey of these algorithms shows that efficiency, accuracy and simplicity of these algorithms can still be improved. However, most of these algorithms focus on particular types of matrices such as positive definite, diagonally dominant, banded and symmetric matrices etc.

In this work a new algorithm is developed for finding inverse of a given matrix. This approach is simpler and efficient than the exiting techniques and is applicable in general irrespective of the structure of the matrix. The manual calculations are straightforward and computer implementation is extremely easy. The memory

DOI: 10.4018/jtd.2010010102
utilization is minimal, i.e., it stores only the original matrix and replaces it gradually by the inverse. The most important and unique features of the algorithm is the ability of finding inversion and determinant in one go.

The rest of the article is organized as follows. In section 2 we are presenting the simple algorithm. To emphasize the simplicity of computer implementation the code is also given in this section. Section 3 demonstrates the use of the algorithm through a numerical example. Section 4 is devoted to comparing computational complexity of the algorithm. In section 5 the performance results of the technique applied to the inversion of various matrices is given. Finally some conclusions are given in section 6.

**A SIMPLE ALGORITHM FOR MATRIX INVERSION**

The algorithm assumes to take a square matrix \( A = [a_{i,j}] \) of dimension \( n \). The inverse is calculated in \( n \) iterations. In each iteration \( p \), all the existing elements \( a_{v,j} \) of \( A \) change to new values \( a'_{v,j} \). After the last iteration i.e. when \( p = n \), \( a'_{v,j} \) will be the elements of the inverse. The determinant of the matrix (denoted by \( d \)) is also calculated iteratively through successive multiplication of the pivot selected in each iteration. In this algorithm the pivots are selected diagonally starting from \( a_{1,1} \) to \( a_{n,n} \). If any pivot is found to be zero i.e., \( a_{p,p} = 0 \) then inverse cannot be calculated. If an inverse is calculated then \( d \) will contain the determinant of \( A \).

A simple improvement to the algorithm is to go to the next diagonal element (in case of zero pivot) and revisit the zero diagonal element later. Probably by that time it would become non zero. Note that in step 7 of the following algorithm \( a'_{v,p} \) on the LHS means that the latest value of the pivot row is to be used in the calculations.

**Step 1:** Let \( p = 0 \), \( d = 1 \);

**Step 2:** \( p \leftarrow p + 1 \)

**Step 3:** If \( a'_{p,p} = 0 \) then cannot calculate inverse, go to step 10.

**Step 4:** \( d \leftarrow d \times a'_{p,p} \)

**Step 5:** Calculate the new elements of the pivot row by:

\[
a'_{i,j} \leftarrow \frac{a_{i,j}}{a_{p,p}}, \text{ where } j = 1, \ldots, n, \quad j \neq p
\]

**Step 6:** Calculate the new elements of the pivot column by:

\[
a'_{i,p} \leftarrow \frac{a_{i,p}}{a_{p,p}}, \text{ where } i = 1, \ldots, n, \quad i \neq p
\]

**Step 7:** Calculate the rest of the new elements by:

\[
a'_{i,j} \leftarrow a'_{i,j} + a'_{i,j} \times a'_{p,j}, \text{ where } i = 1, \ldots, n, j = 1, \ldots, n \text{ and } i \neq j
\]

**Step 8:** Calculate the new value of the current pivot location:

\[
a'_{p,p} \leftarrow \frac{1}{a'_{p,p}}
\]

**Step 9:** If \( p < n \) go to step 2 (\( n \) the dimension of the matrix \( A \)).

**Step 10:** Stop. If inverse exists- \( A \) contains the inverse and \( d \) is the determinant.

**LEMMA:**

Let \( A \) be a non singular matrix of size 2 and \( M_1 \) is obtained from \( A \) through acting suitable matrix transformations. Whereas, \( M_2 = A^{-1} \) is obtained from \( M_1 \) through acting and inserting suitable matrix transformation, then

1. \( F_1 \) the matrix obtained through the 1st iteration of the algorithm coincides with \( M_1 \).
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